Abstract. Mathematical Economics. Kyoto. November 25, 26, 27. 2015

Asymptotic Theory of R-estimators from iid to weakly dependent observations: Cases of one sample problem and simple linear regression with Generalized Lehmann's Alternative Models.

Ryozo Miura
Hitotsubashi University (Professor Emeritus)
Tohoku University (Visiting Professor) Japan

Abstract: Miura&Tsukahara (1993) defined R-estimators for Generalized Lehmann's Alternative Models (or Transformation models) and proved their asymptotic normality under the assumption that the observations are independent and identically distributed. This model includes a usual one sample location models

We extend this result to the case where observations are not iid, but are weakly dependent. This means that a location parameter of a skew symmetric distribution (transformed symmetric distribution) and a skew (or transformation) parameter can be estimated simultaneously, and semi-nonparametrically based on rank statistics. Then, we apply this result to the error terms of a simple linear regression model, where the regression residuals are used to estimate parameters in Generalized Lehmann's Alternative models.

Now, mathematical tools for proving asymptotic normality of R-estimators for weakly dependent cases can be found in Shao & Yu(1993) and Louhichi (2000) which study a weak convergence of an empirical distribution function of weakly dependent random variables

For a regression coefficient in a simple linear regression model with dependent error terms, Koul (1977) proved an asymptotic normality of R-estimators that are derived from rank statistics with bounded score functions. Combining this with our results, it is understood that a simple linear regression model with weakly dependent errors whose identical distribution is skewed (asymmetric) can be treated within the framework of R-estimators all through. This result is good enough for the popular linear (bounded) score function in the class of score functions on the unit interval [0, 1]. However, we are interested in extending the results for the case of unbounded score functions for a wider variety of applications. We will show our recent attempt for this generalization.

Our simple linear regression models may be important in the field of quantitative finance where a simple linear regression model (they call it a "Market model") is used to estimate "Beta" (a regression coefficient), and where, in the real financial market, the

error distributions are often asymmetric and their time series error terms are rather weakly dependent in many cases. So our model fits well to the finance data.

This is a joint work with Professor Dalibor Volny at University of Rouen and Professor Sana Louhichi at University of Grenoble, in France.

References.

[1] Hira L. Koul (1977). "Behavior of Robust Estimators in the Regression Model with Dependent Errors." The Annals of Statistics. Vol. 5, No. 4, 681-699.

[2] Sana Louhichi (2000). "Weak Convergence for Empirical Processes of Associated Sequences." Ann. Inst. Henri Poincaré, Probabilités et Statistiques Vol. 36, No. 5, 547–567

[3] Miura, R. and Tsukahara, H.(1993). "One sample estimation for generalized Lehmann's alternative models." Statistica Sinica. Vol. 3. 83-101.

[4] Miura, R. (2014). "Ippanka sareta Lehmann tairitsukasetsu moderu wo motiita tanjunsenkeikaiki moderu to juni suitei (A simple linear regression model with error terms represented by generalized Lehmann's alternative models and Rank-based estimators)." Syougaku Kenkyuu, Kwansei Gakuin Daigaku, March 2014. pp.89-108. (in Japanese)

[5] Qi-Man Shao and Hao Yu (1993). "Weak Convergence for Weighted Empirical Processes of dependent sequences." The Annals of Probability Vol. 24, No. 4, 2098-2127.