Selfish Routing with Common-Knowledge

Takashi Matsuhisa
Institute of Applied Mathematical Research, Karelia Research Centre, RAS
Pushkinskaya ulitsa 11, Petrozavodsk, Karelia, 185910, Russia
`takashi.matsuhisa@krc.karelia.ru`

This paper starts studying a Bayesian routing problem from the epistemic point of view. We focus on the role of common-knowledge on the users' individual conjectures on the others' selections of channels by giving Bayesian extension of KP-model, which is the network game introduced by Koutsoupias and Papadimitriou [5].

We propose two notions of equilibria, expected delay equilibrium and rational expectations equilibrium, which the former is the profiles of individual conjectures such as each user maximizes his/her own expectations of delay and the latter is the profiles of conjectures such as each user maximizes his/her own expectations of social cost respectively.

We will highlight the epistemic feature of these equilibriums, and we shall show the epistemic condition for the equilibria as below:

Main theorem *If all users commonly know an expected delay equilibrium, then the equilibrium yields a Nash equilibrium in the based KP-model. If they commonly know a rational expectations equilibrium, then the equilibrium yields a Nash equilibrium for social cost in it.*

Further we will extend the notion of price of anarchy to rational expectations equilibriums, and give its upper bounds for typical social cost functions.

Garing et al [3] is the first paper in which Bayesian Nash equilibrium is treated. They analysis Bayesian extension of routing game specified by the type-space model of Harsanyi [4] as information structure, and they collected several results: (1) the existence and computability of pure Nash equilibrium, (2) the property of the set of fully mixes Bayesian Nash equilibria and (3) the upper bound of the price of anarchy for specific types of social function associated with Bayesian Nash equilibria.

In this paper we shall modify their model by adopting arbitrary partition structure following Aumann [1] instead of the type-space model. The merit of adopting information partition structure lies not only in getting the close connection to computational logic (Fagin et al [2]) but also in increasing the range of its applications in various fields.

Finally we have to remark on common-knowledge assumption. The assumption plays essential role in the above theorem if there are more than two users. In fact, for two users case the theorem is still true without common-knowledge assumption, however for 3 users case it cannot hold without the assumption.

We know it is actually very strong assumption, because common-knowledge is introduced by the infinite regress of interactions among individual knowledge. So we would like to remove out it in our framework. There seems to be several ways to improving this point, here I would recommend adopting the communication process introduced by Parikh and Krasucki [6] replacing common-knowledge.

[1] Aumann, R. J.: Agreeing to disagree. *Annals of Statistics* **4** (1976), 1236–1239.
[2] Fagin, R., Halpern, J. Y., Moses , Y. and Vardi, M.Y.: *Reasoning about Knowledge*. MIT Press, Cambridge, Massachusetts, London, England, 1995.
[3] Garing, M., Monien, B. and Tiemann, K.: Selfish Routing with Incomplete Information. *Theory Compt Syst* **42** (2008) pp.91-130.
[4] Harsanyi, J.C.: Games with incomplete information played by Bayesian players, I, II, III. Management Science **14**, pp.159-182, 320-332, 468-502 (1967)
[5] Koutsoupias, E. and Papadimitriou, C. H.: Worst-case equilibria. In: Meinel, C. and Tison, S. (ends) Proceedings of the 16th Internatinal Symposium of Theoretical Aspect of Computer Science. Lecture Notes in Computer Science, **vol.** 1563, pp.404-413. Springer (1999)
[6] R. Parikh and P. Krasucki: Communication, consensus, and knowledge. *Journal of Economic Theory* **52** (1990) pp.78–89.