REAL ALGEBRAIC METHODS IN OPTIMIZATION

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Recently, studies on optimization theory and real algebraic geometry came to a new stage. By discovery of new algorithms, we can now obtain a *global optimal value* of polynomial optimization problems under a moderate assumption, which before could not be solved in practice but only in theory. Surprisingly, the behaviors of the algorithm highly depend on deep and abstract results from real algebraic geometry. In recent years, optimization theorists have been finding various applications of real algebraic geometry to optimization. On the other hand, real algebraic geometers are exposed to questions coming from practice which open new surprising theoretical research directions.

In this talk, we overview how real algebraic methods can be applied in optimization. After showing several simple examples, we explain theorems of alternatives nonconvex polynomial systems. Then an algorithm for obtaining global optimal value of polynomial optimization problem is introduced. It generates a sequence of semidefinite programmings. We discuss relationships between their strong duality and real ideals which are important objects in real algebraic geometry. Although the algorithm needs infinitely many semidefinite programmings in principle, optimality conditions at global optimizers ensure that finite number of semidefinite programmings are enough to obtain global optimal value. This phenomenon is related to the question whether a positive polynomial can be written as sums of squares of polynomials.

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