

Fast Bellman Iteration: An Application of Legendre-Fenchel Duality to Infinite-Horizon Dynamic Programming in Discrete Time

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In this paper, we apply tools from convex analysis to the solution of dynamic programming problems. We propose an algorithm, which we call "Fast Bellman Iteration" (FBI), to compute the value function of an infinite-horizon dynamic programming problem in discrete time. FBI is applicable to a wide range of multidimensional dynamic programming problems with concave return functions (or convex cost functions) and linear (or affine) constraints, enabling one to compute the value function in an extremely efficient way.

In this algorithm, the Legendre-Fenchel transform of the return function is first computed, and then an arbitrary initial function is repeatedly updated according to a simple algebraic rule. The sequence of the resulting functions is guaranteed to converge, and the Legendre-Fenchel transform of the limiting function is the value function. The Bellman operator, which is computationally expensive and difficult to characterize in standard form, becomes a tractable scaling and addition operation on Legendre-Fenchel transforms.