Upper-convergent Dynamic Programming

Hiroyuki Ozaki
Faculty of Economics
Keio University

December 2, 2014

Abstract

This paper shows that dynamic programming techniques are applicable to time-separable utility functions under the assumption of upper-convergence. That is, we show the existence of the value function, that it is characterized as the greatest solution to Bellman's equation and that recursive optimality implies optimality. The concept of upper-convergence was first developed by Streufert (1990) and then by Ozaki and Streufert (1996) in a stochastic context. It essentially requires that even if a consumption path in the very far future is replaced by the pure accumulation path defined through the production function, the value of utility function should not be changed. In particular, it is shown that all of the discounted nonnegative returns considered by Blackwell (1965), the nonpositive returns considered by Strauch (1966) and the discounted logarithmic returns can be handled by the single concept of upper-convergence. Furthermore, we develop a convenient approximation method for the value function based on the upper-convergence.