On the Lebesgue property and related regularities of monotone convex functions on Orlicz spaces

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Many problems in financial mathematics involve monotone convex functions of measurable functions including especially convex risk measures, expected/robust utility functionals and their variants. The regularities of those functionals w.r.t. the natural order structure often play key roles. In this talk, we characterize the so-called Lebesgue property (continuity w.r.t. the dominated a.s. convergence) in terms of other topological regularity properties motivated by a James-type result by [2, 1] which asserts that for any convex risk measure on L^∞ having the Fatou property (or equivalently the σ-additive dual representation), there is equivalence between (1) the Lebesgue property, (2) the weak compactness of all the sublevel sets of the conjugate, and (3) the attainment of the supremum in the dual representation. Each of equivalent properties has importance in application, and the implication (3) $\Rightarrow$ (2) may be viewed as a partial generalization of perturbed James' theorem. Recently, [3] obtained a similar equivalence for risk measures on certain class of Orlicz spaces.

We provide a couple of generalizations of this result, with slightly different a priori assumptions and the choice of penalty function, for monotone convex functions on lattice ideals (solid vector subspaces) of L^0, including especially all Orlicz spcaes, which improve the one by [3], with a much simpler proof, and unifies several other related results. We then discuss applications and implications in financial mathematics.

References

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