

# Bounded response and Arrow's impossibility

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## Abstract

We propose a new axiom called *bounded response*, and discuss relations to Arrow's impossibility theorem. Two preferences are said to be *adjacent* if they differ only in an ordering of a pair of alternatives. Two preference profiles are adjacent if only one agent's preferences are adjacent and each other's preferences are the same. *Bounded response* requires that for each pair of adjacent preference profiles, the social welfare function should assign a pair of adjacent or equal social preferences. This axiom is, thus, weaker than *independence of irrelevant alternatives* adopted in Arrow's impossibility theorem. A social welfare function satisfies *adjacency-extended unanimity* if it satisfies unanimity (the social preference is the same as an agent's whenever each agent has the same preference) and if each agent has one of two preferences  $R, R'$  which are adjacent, the social preference is also  $R$  or  $R'$ . We show that *bounded response* together with *adjacency-extended unanimity* imply dictatorship whenever there are four or more alternatives. If there are only three alternatives, there exist non-dictatorial social welfare functions satisfying these two axioms, but there exists a unique agent  $i$  such that if  $i$  knows the other agents' preferences,  $i$  can realize any preference as the social preference by choosing  $i$ 's preference appropriately. In the proof, we employ a new technique inspired by the fundamental group theory in algebraic topology.