On Pure-Strategy Equilibria in Games with Correlated Information*

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Abstract: Aumann (1974) introduced the notions of *secret* and *objective* events in a setting with *subjective* information and beliefs, but a decisive and celebrated example of Radner-Rosenthal (1982) questioned the hypotheses of the result that the set of independent objective pure-strategy equilibrium payoffs of a suitably-formulated incomplete-information game coincides with the set of mixed-strategy equilibrium payoffs of the original complete information game of Nash (1950, 1951). We present a two-player game with information modeled as a subset in the product of the extended Lebesgue interval, as proposed in Khan-Zhang (2012), and show that the sub-σ-algebra of a player's secret events is *rich* enough to adequately respond to this criticism, and to rescue Aumann's original motivation towards a descriptive theory of pure-strategy equilibrium in games with correlated information. We also show that a *saturated* information structure, as emphasized by Keisler-Sun (2009), is *necessary* to guarantee the existence of pure-strategy equilibrium in a simple subclass of such games. Our results generalize beyond the toy-model, but we emphasize the simplest setting with many illustrative examples. 169 words

Key Words: Correlated information games, Lebesgue extension, saturated probability spaces, secret event, subjective (objective) pure-strategy equilibrium

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