

The Bang-Bang, Purification and Convexity Principles in Infinite Dimensions: Additional Characterizations of the Saturation Property*

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Abstract

The authors apply their recent work on the Lyapunov theorem in locally convex Hausdorff spaces to the bang-bang principle for control systems in infinite dimensions. They show that the bang-bang principle holds for every integrably bounded, measurable, weakly compact convex-valued multifunction if and only if the underlying measure space is saturated. They also demonstrate the equivalence of the bang-bang principle to what is termed the purification and convexity principles. Applications to variational problems with integral constraints are indicated.

Key Words: Lyapunov theorem, Dvoretzky-Wald-Wolfowitz theorem, saturated measure space, bang-bang principle, relaxed control, purification principle, convexity principle.

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