Asset Demand and Ambiguity Aversion

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Abstract

In many uncertain situations, it is extremely difficult or merely impossible to estimate the distributions of possible outcomes, because there are only few samples or the underlying mechanism is complex. When the asset returns depend on these uncertainties, it is unreasonable to assume that investors have expected utility functions, because, to calculate expected utilities, it would be necessary to know the distributions of asset returns, which are, in fact, unknown to investors. In the presence of such *ambiguous* uncertainties, it is more reasonable to use utility functions that are averse not only to risk but also to ambiguity. In this paper, we study the optimal portfolio choice problem of an investor who exhibits ambiguity aversion. More specifically, in the traditional single-period setting, we consider an investor who has a utility function in the class proposed by Klibanoff, Marinacci, and Mukerji (2005). By controling the degree of ambiguity aversion, we investigate how the optimal portfolio changes as the degree of ambiguity aversion increases, while the degree of risk aversion remains constant.

Our analysis specializes in the case where the degree of absolute risk aversion and the degree of ambiguity aversion is constant (independent of consumption levels) and the asset returns are normally distributed. The ambiguity lies in the means of the asset returns, which are themselves random variances. Our first main result, Theorem 1, is stated in terms of a matrix, denoted by Q, that roughly measures the ratio of the variance of asset returns due solely to ambiguity to the total variance of these asset returns. It represents the optimal portfolio as a linear combination of the eigenvectors of Q, with the associated coefficients depending on the degree of ambiguity aversion in excess of the degree of risk aversion. Our second main result, Theorem 2, decomposes an expected-utility maximizer's optimal portfolio into two portfolios, one remaining and the other vanishing as the degree of ambiguity aversion in excess of the degree of risk aversion goes to infinity, and characterizes each of the two as an expected-utility maximizer's optimal portfolio for an appropriately chosen pair of a mean vector and a covariance matrix. This result gives an economic interpretation of the optimal portfolio and turns out to be useful in applications.

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