On the resource allocation problems with interpersonal comparisons of welfare: Extended preference approach revisited

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Introduction

- This paper introduces the extended preference approach studied in the literature of social welfare functionals into the pure exchange economy model.
- We show that the leximin criterion, the equal income Walras rule, and price indicator types of extended preferences are tied up together; any two of the three imply the rest. This tied-up relation derives an axiomatization of the equal income Walras rule using the leximin fairness.

Exchange Economies 1/3

- The notation for vector inequalities： $\geq, >, \gg$
- The set of real numbers：R
- $\Delta^{\ell} := \{p \in R_{+}^{\ell} \mid \sum_{i=1}^{\ell} p_i = 1\}$, $int. \Delta^{\ell} := \{p \in R_{++}^{\ell} \mid \sum_{i=1}^{\ell} p_i = 1\}$,
- The set of agents：$N = \{1,2, \dots, n\}$
- The set of private goods：$L = \{1,2, \dots, \ell\}$
- All agents have the same consumption set：R_{+}^{ℓ}

Exchange Economies 2/3

- Agent i's consumption：$z_i = (z_{i1}, \dots, z_{i\ell}) \in R_+^{\ell}$
- Allocation ：$z = (z_1, \dots, z_n) \in R_+^{n\ell}$
- Total endowment of the economy：$\Omega \in R_{++}^{\ell}$
- z is feasible if $\sum_{i \in N} z_i \leq \Omega$. Z be the set of all feasible z.
- Let $\succcurlyeq_i$ be agent i's preference on R_+^{ℓ} and continuous, convex, and monotonic on R_+^{ℓ}. Q is the set of all $\succcurlyeq_i$.

Exchange Economies 3/3

- A list $\succcurlyeq = (\succcurlyeq_i)_{i \in N} \in Q^n$, is called a profile.
- A feasible z is Pareto optimal if there are no other feasible ones z' with $z'_i \succ_i z_i$ for all $i \in N$. $[PO(\succcurlyeq), PO(\succcurlyeq, p)]$

  ↑supporting price
- A feasible allocation z is an equal income Walrasian allocation if there exists some $p \in int.\Delta^\ell$ such that $z_i \succcurlyeq_i x$ for all $x \in R^\ell_+$ with $px \leq p\left(\frac{\Omega}{n}\right)$. $[EIW(\succcurlyeq), EIW(\succcurlyeq, p)]$

  ↑equilibrium price

Extended Preferences 1/3

- The notion of extended preferences is based on the principle of extended sympathy mentioned by Arrow (1963) and initiated by Suppes (1966) and Sen (1970).
- The basic idea is that a hypothetically existing ethical observer compares the welfare of different persons from a social point of view while respecting (or sympathizing with) their subjective preferences.

Extended Preferences 2/3

- An extended preference $\succcurlyeq_E$ created from $\succcurlyeq \in Q^n$ is a complete and transitive binary relation on $R_+^\ell \times N$.

◆$(x, i) \succcurlyeq_E (y, j)$ ：being agent i with consumption x is at least as well off as being agent j with consumption y.

- A set of continuous utility functions $(u_i)_{i \in N}$ represents $\succcurlyeq_E$ if $(x, i) \succcurlyeq_E (y, j) \Leftrightarrow u_i(x) \geq u_j(y)$ for all $i, j \in N$ and all $x, y \in R_+^\ell$.

Extended Preferences 3/3

$E(\succcurlyeq)$ be the set of $\succcurlyeq_E$ satisfying E.1, E.2 and E.3.

- E.1. For any $i \in N, x \succcurlyeq_i y \Leftrightarrow (x, i) \succcurlyeq_E (y, i)$ for all $x, y \in R_+^\ell$.
- E.2. $(x, i) \succcurlyeq_E (0, j)$ for all $i, j \in N$ and all $x \in R_+^\ell$.
- E.3. $\succcurlyeq_i$ has representation.

✓E.2 reflects our intuition that as long as other conditions are equal, people without wealth are the most miserable in the world. Note that E.2 implies $(0, i) \sim_E (0, j)$ for all $i, j \in N$.

Leximin Criterion 1/3

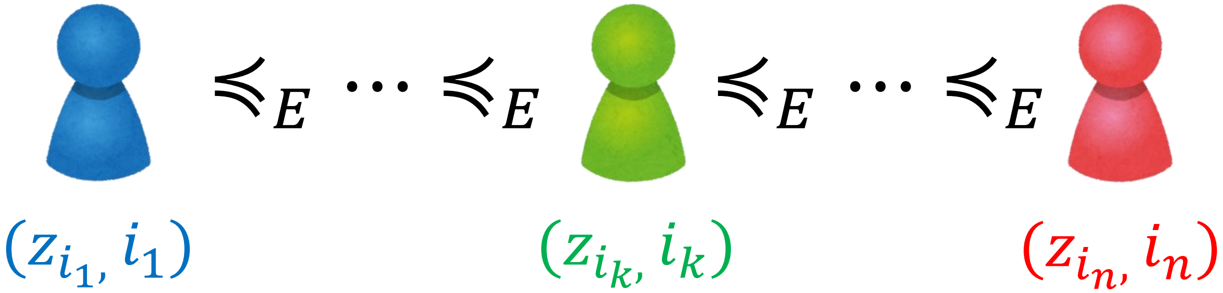
The leximin criterion, a lexicographic extension of the difference principle of Rawls(1971).

- THE VEIL OF IGNORANCE: *First of all, no one knows his place in society, his class position or social status... The persons in the original position have no information as to which generation they belong.* (Sec.24)

- SECOND PRINCIPLE: *Social and economic inequalities are to be arranged so that they are both: (a)to the greatest benefit of the least advantaged, consistent with the just savings principle, and (b)attached to offices and positions open to all under conditions of fair equality of opportunity.* (Sec.46)

Leximin Criterion 2/3

Given an allocation z, we arrange all (z_i, i) in ascending order s.t. $(z_{i_1}, i_1) \preccurlyeq_E (z_{i_2}, i_2) \preccurlyeq_E \cdots \preccurlyeq_E (z_{i_{n-1}}, i_{n-1}) \preccurlyeq_E (z_{i_n}, i_n)$, where tie is broken arbitrarily.

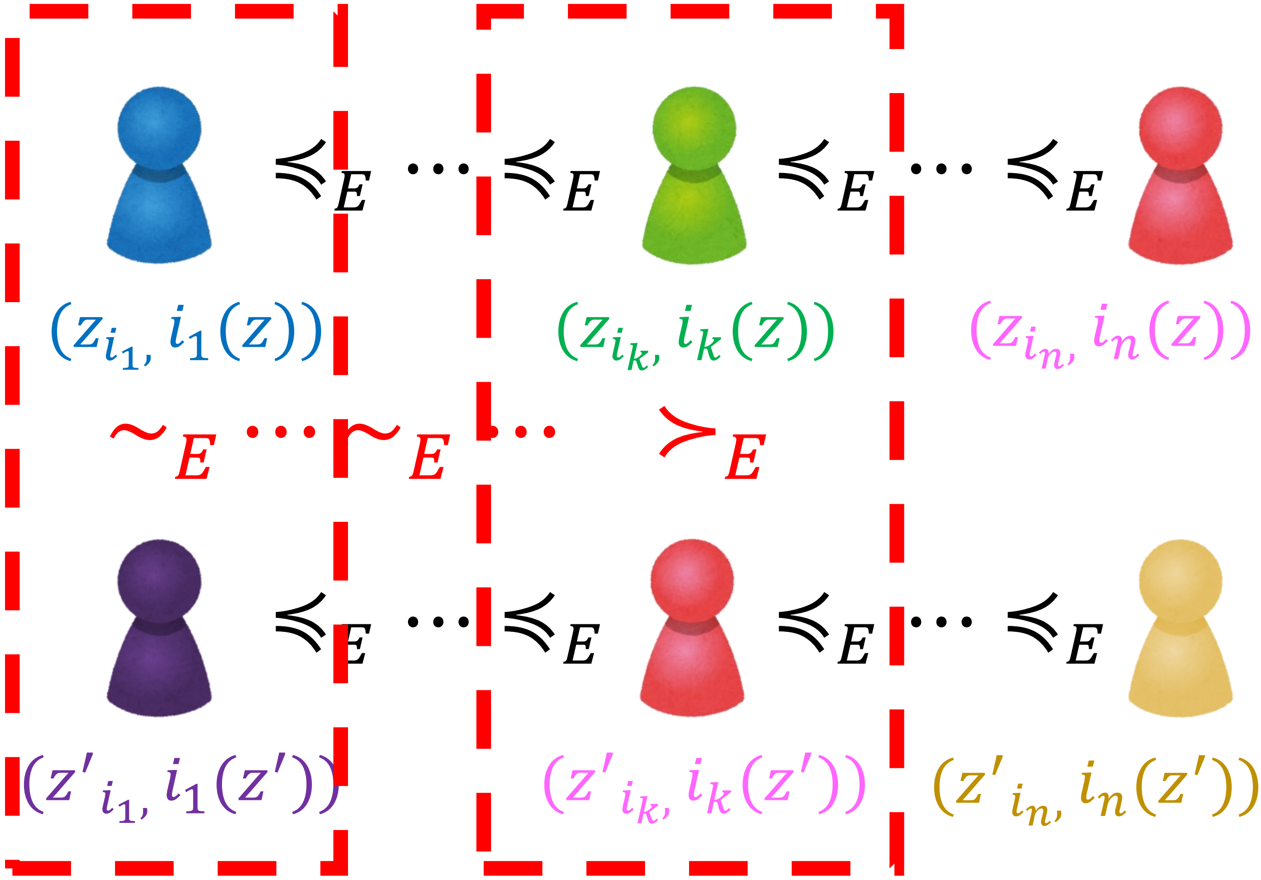


The agent i_k $(k = 1, \dots n)$ is the k-th worst off agent in z.

Denote i_k by $i_k(z)$. Let us consider lexicographic order $\geq_{L(E)}$.

Leximin Criterion 3/3

- A feasible z is leximin equitable if there are no other feasible z' with $z >_{L(E)} z'$. $[LME(\succcurlyeq)]$



$z >_{L(E)} z' \Leftrightarrow \exists k \in \{1, \ldots, n\}$ s.t.
$(z_{i_\tau(z)}, i_\tau(z)) \sim_E (z'_{i_\tau(z')}, i_\tau(z'))\ \forall \tau \in \{1, \ldots, k-1\}$
and $(z_{i_k(z)}, i_k(z)) \succ_E (z'_{i_k(z')}, i_k(z'))$

Price indicator type

An extended preference $\succcurlyeq_E$ is a price indicator type at z and p if $z \in PO(\succcurlyeq, p)$, and for any $i, j \in N$ and any $x \in R_+^\ell$, $(x, j) \succcurlyeq_E (z_i, i)$ implies $px \geq pz_i$, and $(x, j) \succ_E (z_i, i)$ implies $px > pz_i$.

✓The welfare ranking at z is completely determined with p.

✓Example: $(x, i) \succcurlyeq_E (y, j) \Leftrightarrow \min\{pq\colon q \sim_i x\} \geq \min\{pq\colon q \sim_j y\}$ is a price indicator type at z and p if $z \in PO(\succcurlyeq, p)$.

Lemma 4

Take $z \in Z, p \in int.\Delta^{\ell}, \succcurlyeq \in Q^n$, *and* $\succcurlyeq_E \in E(\succcurlyeq)$*arbitraly.*
Any two of the three statements below imply the rest.
Therefore, they are equivalent to each other.

(1) $z \in EIW(\succcurlyeq, p)$

(2) $z \in LME(\succcurlyeq_E)$

(3) $\succcurlyeq_E$ *is a price indicator type at* p *and* z*.*

How to extend preferences 1/2

Given $\succcurlyeq \in Q^n$ and $z \in Z$, construct $D(\succcurlyeq, z) \subset E(\succcurlyeq)$ according to **D.1** and **D.2**.

D.1 If $\succcurlyeq = (\succcurlyeq, \dots, \succcurlyeq) \in Q^n$, $D(\succcurlyeq, z) = \{\succcurlyeq_E\}$ for any $z \in Z$, where $\succcurlyeq_E$ satisfies $(x, i) \succcurlyeq_E (y, j) \Leftrightarrow x \succcurlyeq y$ for any $i, j \in N$ and $x, y \in R_+^\ell$.

D.2 For any smooth $\succcurlyeq, \succcurlyeq' \in Q^n$ and any $z \in Z \cap R_{++}^{n\ell}$, if for every i, MRS between any two goods at z_i remain unchanged across from $\succcurlyeq_i$ to $\succcurlyeq_i'$, then welfare rankings at z are same.

How to extend preferences 2/2

D.1 is a self-evident truth, requiring that if every agent has the same preference, that preference should be the only extended preference.

D.2 is a requirement of informational economization in making interpersonal comparisons of welfare, which claims that the welfare ranking at an allocation be determined only by local information of individual preferences around the allocation, independently from preferences about allocations far away.

- cf. Local Independence condition in Nagahisa(1991)

Social choice rules 1/2

Let D be the domain, a nonempty subset of Q^n.

A rule F is a mapping that associates with each $\succcurlyeq \in D$ a non-empty subset of Z. It decides $F(\succcurlyeq)$ comparing the welfare of different agents. (It uses *some* extended preferences created from $\succcurlyeq$, but not necessarily *all*.)

Let us call $\bigcup_{(\succcurlyeq, z) \in D \times Z} D(\succcurlyeq, z)$ the extended domain.

Social choice rules 2/2

1. A rule F satisfies Leximin Justification Possibility $(LMJP)$ if $F(\succcurlyeq) \subset \{z \in Z : z \in \bigcup_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E)\}$ for any $\succcurlyeq \in D$.

✓If we want to select z, at least one extended preference must make it leximin equitable.

2. A rule F meets Leximin All Unaminity$(LMAU)$ if $\{z \in Z : z \in \bigcap_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E)\} \subset F(\succcurlyeq)$ for any $\succcurlyeq \in D$.

✓If z becomes leximin equitable for all extended preferences, then we must select it.

Theorem 2

$LME^{\exists}(\succcurlyeq) = \{z \in Z : z \in \bigcup_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E)\}$

$LME^{\forall}(\succcurlyeq) = \{z \in Z : z \in \bigcap_{\succcurlyeq_E \in D(\succcurlyeq, z)} LME(\succcurlyeq_E)\}$

Let F and D be a rule and an extended domain, respectively. Suppose that $LMJP$ and $LMAU$ are defined on D. The statements below are equivalent.

(1) There exits some D such that F is the only rule satisfying $LMJP$ and $LMAU$.

(2) There exits some D such that $LME^{\forall}(\succcurlyeq) = F(\succcurlyeq) = LME^{\exists}(\succcurlyeq)$ for all $\succcurlyeq \in D$.

$z_i \neq 0$ for all i and all $z \in F(\succcurlyeq)$

(3) F satisfies Pareto Optimality and <u>Non-Zero Rationality.</u>

Theorem 3

Let D *be an extended domain. The statements below are equivalent.*

(1) EIW *is the only rule satisfying* $LMJP$ *and* $LMAU$ *defined on* D.

(2) D *consists of price indicator types.*

Summing up results

The leximin criterion (LM), equal income Walras rule (EIW), and price indicator type of extended preferences (PIT) are inseparably tied together.

(1) LM and PIT imply EIW.

(2) LM and EIW imply PIT.

(3) EIW and PIT imply LM.

Conclusion 1/2

Hammond (1991) stated the following about interpersonal comparisons of utilities (ICUs):

But if it is easier to think what is a good social welfare ordering, rather than how to make ICUs, why should we not start with the ordering and have it reveal the ICUs, instead of starting with ICUs and trying to derive a social ordering? Especially if it is not at all clear anyway how to incorporate ICUs into a social ordering even if we believe we have made securely founded and ethically relevant interpersonal comparisons of both utility levels and utility differences.

Conclusion 2/2

Though only dealing with exchange economies, our result answers Hammond’s question. The fact that people accepted the rules corresponds to the fact that people agreed with making welfare comparisons based on the rules because each person receives their utility according to the rules.

According to Hammond’s argument, accepting EIW and the leximin fairness as valuable is equivalent to regarding welfare comparisons of D.1 and D.2 as correct.

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