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Resource allocation problems with interpersonal welfare comparison: An axiomatization of the impartial Walras rule

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1 The purpose

- Applying extended sympathy approach to resource allocation problems
- An axiomatization of the impartial Walras Rule

2 Nagahisa (1991)

- The Walras rule is the unique rule satisfying individual rationality, Pareto optimality, nondiscrimination, and local independence
- The same axiomatization leads to the impartial Walras rule, not to the Walras rule if interpersonal comparisons of welfare are admitted.

3 Extended sympathy approach

- Extended preference; Arrow (1961), Suppes (1966), Sen (1970).

$$(x, i) \succsim_E (y, j)$$

Being in i 's position in social state x is at least as good as being at j 's position in social state y

4 Extended sympathy approach

- From the viewpoint of impartial observer.
- Putting oneself in another shoes$\Longrightarrow$the possibility of interpersonal comparisons of welfare
- The departure from from Arrow's impossibility theorem: relaxing IIA
- The utilitarian rule and the leximin rule, Hammond(1976), Roberts(1980a,b), Nagahisa and Suga(1996).

5 Application to resource allocation problems

- To justify market mechanism by extended sympathy approach.
- Adam Smith problem: public interest vs. private interest

6 Exchange economies

$N = \{1, 2, ..., n\}$: the set of agents

$L = \{1, 2, ..., l\}$: the set of private goods

R^l_+ : consumption set

$z_i = (z_{i1}, ..., z_{il}) \in R^l_+$: agent i's consumption

$z = (z_1, ..., z_n) \in R^{nl}_+$: an allocation

$\omega_i \in R^l_{++}$: i's initial endowment

$\omega = (\omega_1, ..., \omega_n) \in R^{nl}_{++}$: the endowment profile

An allocation z is feasible if $\sum_{i \in N} z_i = \sum_{i \in N} \omega_i$

7 Exchange economies

$\succsim_i$: i's preference on R^l_+.

$\succsim = (\succsim_i)_{i \in N}$: a (preference) profile

D : the domain, the set of profiles.

8 $D = L \cup Q^n$

$$\Delta^l := \{p \in R^l_+ : \sum_{i=1}^{l} p_i = 1\},$$

$$int.\Delta^l := \{p \in R^l_{++} : \sum_{i=1}^{l} p_i = 1\}.$$

- $\succcurlyeq^p = (\succcurlyeq^p_i)_{i \in N} \in L \Longleftrightarrow \exists p \in int.\Delta^l$ s. t. $\forall i$, $x \succcurlyeq_i y \Longleftrightarrow px \geq py \ \forall x, y \in R^l_+$.

9 $D = L \cup Q^n$

- $Q^n = \overbrace{Q \times \cdots \times Q}^{n}$
- $\succsim_i \in Q$ if
- $\succsim_i$ is continuous and convex on R^l_+ and

  continuously differentiable on R^l_{++}.
- $x \gneq y$ implies $x \succsim_i y$, and if in addition $x \in R^l_{++}$, this implies $x \succ_i y$.
- $\forall\ x \in R^l_{++}$, $\{y \in R^l_{++} : y \succsim_i x\} \subset R^l_{++}$.

10 Allocations

$z \in Z$ is

- Pareto optimal $\iff$ No feasible allocation z' such that $z'_i \succcurlyeq_i z_i \ \forall i$ and $z'_i \succ_i z_i, \ \exists i$
- individually rational $\iff z_i \succcurlyeq_i \omega_i, \forall \ i$
- a Walrasian allocation $\iff \exists p \in int.\Delta^l$ s. t. $\forall \ i \in N$, $z_i \succcurlyeq_i x_i \ \forall x_i \in R^l_+$ s. t. $px_i \leq p\omega_i$.

11 Allocations

- $PO(\succcurlyeq)$:the set of Pareto optimal allocations
- $IR(\succcurlyeq)$: the set of individual rational allocations
- $W(\succcurlyeq)$: the set of Walrasian allocations.

12 Extended Preferences

A hypothetically existing ethical observer compares the welfare of different persons from a social point of view while respecting (or sympathizing with) their subjective preferences.

13 Extended Preferences

- Given $\succsim \in D$, an extended preference $\succsim_E$ generated from $\succsim$ is a complete and transitive binary relation on $R^l_+ \times N$.

$$(x, i) \succsim_E (y, j)$$

- "being agent i with consumption x is at least as well off as being agent j with consumption y."

14 Extended Preferences

- The axiom of identity:
  $x \succsim_i y \Longleftrightarrow (x, i) \succsim_E (y, i) \ \forall x, y \in R^l_+ \ \forall i.$

15 Extended Preferences

- Example 1

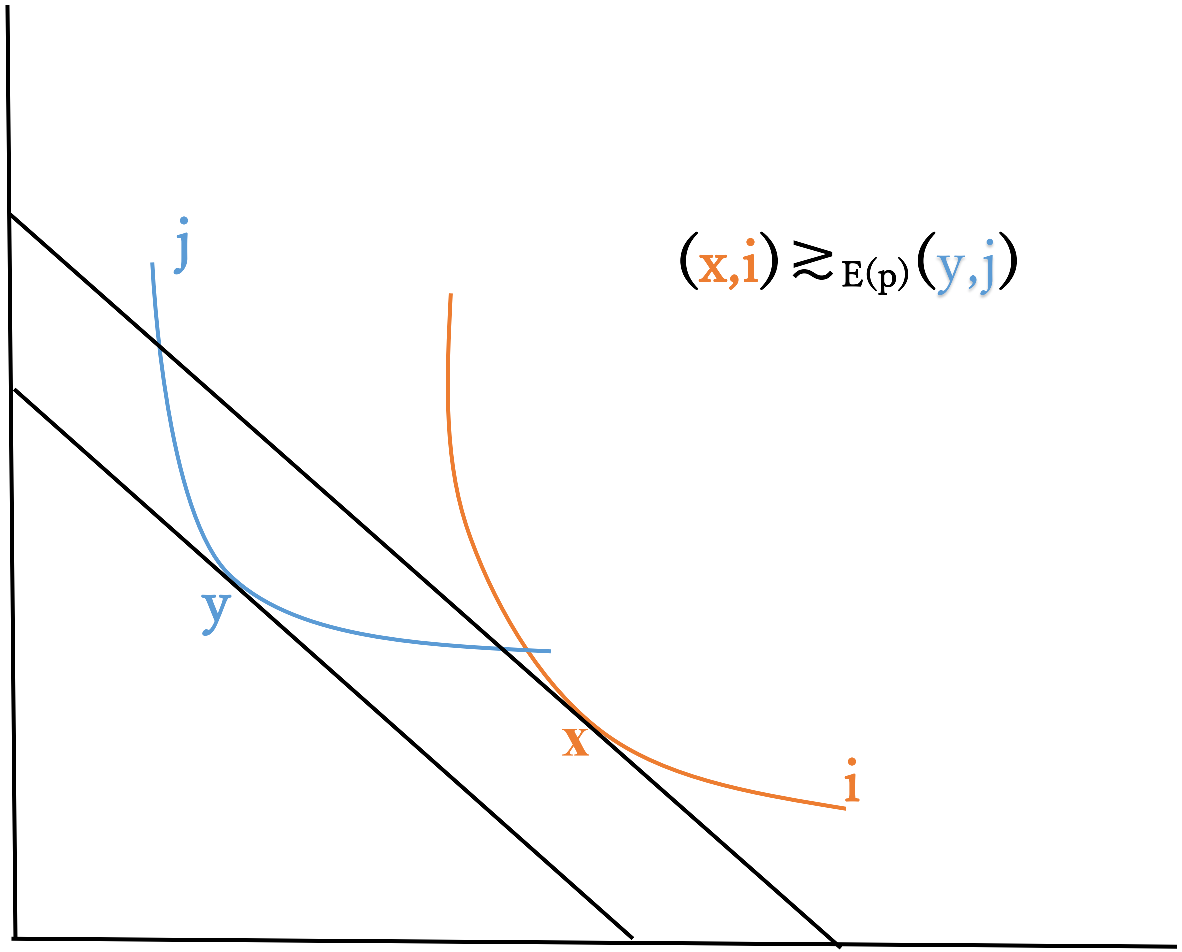
  Let $\succsim \in D$ and $p \in int.\Delta^l$ be given.

  $(x, i) \succsim_{E(p)} (y, j) \Longleftrightarrow \min\{pq : q \sim_i x\} \geq \min\{pq : q \sim_j y\}$.

- Example 2

  If $\succsim^p \in L$, then $\succsim^p_{E(p)}$ reduces to a simple form as follows.

  $(x, i) \succsim^p_{E(p)} (y, j) \Longleftrightarrow px \geq py$.


j
$(x,i) \succsim_{E(p)} (y,j)$
y
x
i

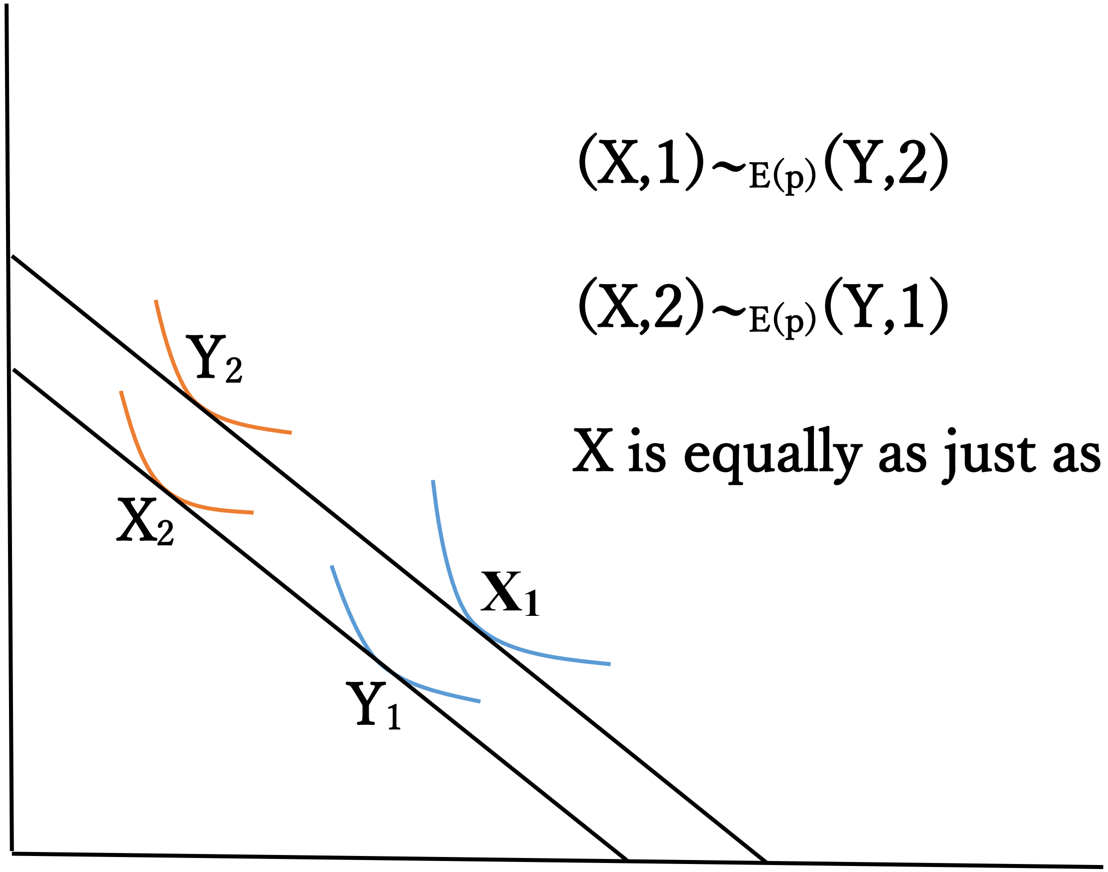
16 Suppes criterion

Π: the set of permutation on N.

Given z and $\pi \in \Pi$, z^π be s. t. $z^\pi_i = z_{\pi(i)}, \forall i$.

17 Suppes criterion

- z is at least as just as z' for $\succsim_E$ if and only if $\exists \pi \in \Pi$ s. t. $(z_i, i) \succsim_E (z'_{\pi(i)}, \pi(i))\ \forall i$
- z is more just than z' for $\succsim_E$ if and only if z is at least as just as z' with $(z_i, i) \succ_E (z'_{\pi(i)}, \pi(i))\ \exists i$
- z is equally as just as z' for $\succsim_E$ if and only if $\exists \pi \in \Pi$ s. t. $(z_i, i) \sim_E (z'_{\pi(i)}, \pi(i))\ \forall i$.



$(X,1) \sim_{E(p)} (Y,2)$

$(X,2) \sim_{E(p)} (Y,1)$

X is equally as just as Y for $\succsim_{E(p)}$

18 Suppes criterion

A feasible allocation z is Suppes equitable for $\succcurlyeq_E$ if and only if there are no feasible allocations z' that are more just than z for $\succcurlyeq_E$.

19 Suppes criterion

- $SE(\succcurlyeq_E)$: the set of Suppes equitable allocations for $\succcurlyeq_E$.
- $IMR(\succcurlyeq_E)$: the set of impartial rational allocations that are at least as just as ω for $\succcurlyeq_E$.
- $SE(\succcurlyeq_E) \subset PO(\succcurlyeq)$ and $IR(\succcurlyeq) \subset IMR(\succcurlyeq_E)$ for all $\succcurlyeq_E$. Note $SE(\succcurlyeq_{E(p)}) \neq \emptyset$.

20 Rules

$f : D \longrightarrow Z$: a rule.

$$\succcurlyeq \in D \xrightarrow{\text{extend preferences}} D(\succcurlyeq) \xrightarrow{f} Z$$

$D(\succcurlyeq)$: a nonempty subset of $\succcurlyeq_E$

$$\succcurlyeq \in D \xrightarrow{f} Z \text{ (Nagahisa1991)}$$

21 Rules

- $\bigcup_{\succcurlyeq \in D} D(\succcurlyeq)$: the extended domain.
- $D(\succcurlyeq)$ satisfies

  D.1. $\forall \succcurlyeq^p \in L$, then $D(\succcurlyeq^p) = \{\succcurlyeq^p_E\}$.

  D.2.$\forall \succcurlyeq \in D$, $\forall \succcurlyeq_E \in D(\succcurlyeq)$, $(x, i) \succcurlyeq_E (0, j)$ $\forall i, j \in N \ \forall x \in R^l_+$.

  D.3.$\forall \succcurlyeq \in D$, $\forall p \in int.\Delta^l$, $\succcurlyeq_{E(p)} \in D(\succcurlyeq)$.

  $D(\succcurlyeq)$ is well defined.

22 Walras rule vs. impartial Walras rule

$$\succcurlyeq \in D \quad \xrightarrow{W} \quad W(\succcurlyeq, \omega)$$
$$\succcurlyeq \in D \quad \xrightarrow{IW} \quad \bigcup_{\pi \in \Pi} W(\succcurlyeq, \omega^{\pi})$$

$W(\succcurlyeq, \omega^{\pi})$: The set of Walrasian allocations when all agents' endowment are pemuted through π:
If ω_i is equal across all i, then $W = IW$.

23 Axioms

This paper		Nagahisa (1991)
Suppes Nondiscrimination	$\Longleftrightarrow$	Utility Nondiscrimination
Suppes Equity	$\Longleftrightarrow$	Pareto Optimality
Impartial Rationality	$\Longleftrightarrow$	Individual Rationality
Extended Local Independence	$\Longleftrightarrow$	Local Independence

- A rule f satisfies SN if
  $\forall \succcurlyeq \in D$, and $\forall\ z, z' \in Z$, z is equally as just as z' $\forall \succcurlyeq_E \in D(\succcurlyeq)$,
  then $z \in f(\succcurlyeq) \Longleftrightarrow z' \in f(\succcurlyeq)$.
- A rule f satisfies UN if
  $\forall \succcurlyeq \in D$, and $\forall\ z, z' \in Z$, $z_i \sim_i z'_i\ \forall i$,
  then $z \in f(\succcurlyeq) \Longleftrightarrow z' \in f(\succcurlyeq)$.

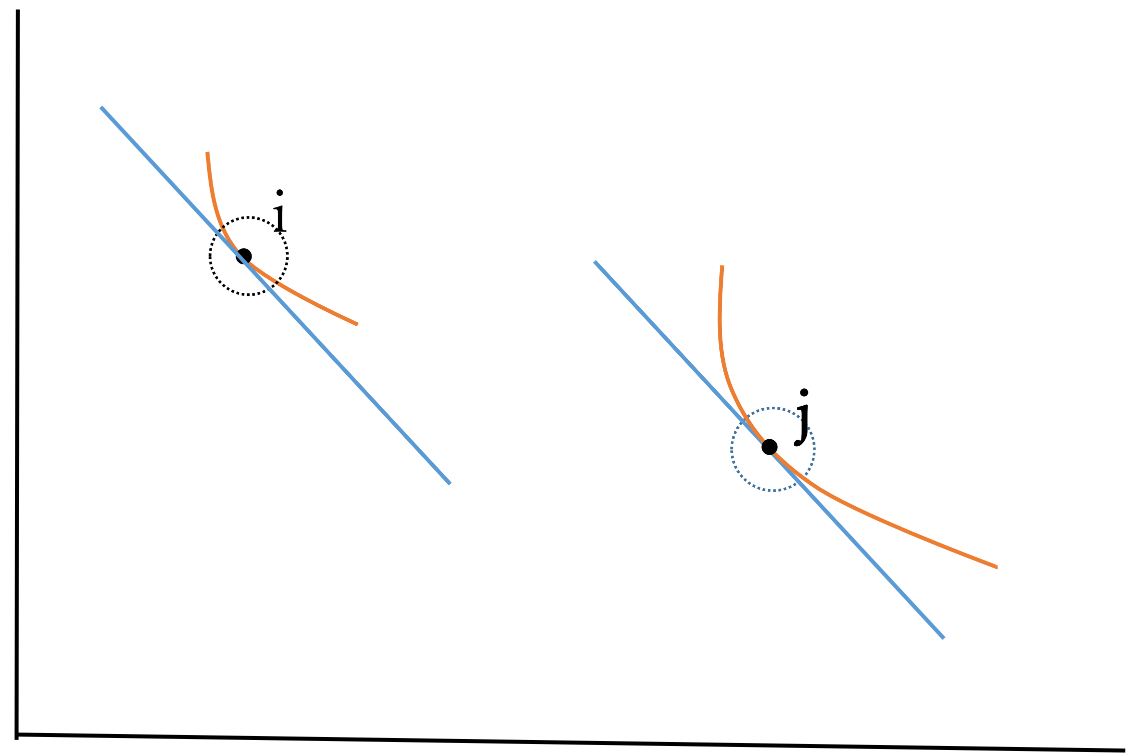
- A rule f satisfies SE if $f(\succcurlyeq) \subset \bigcup_{\succcurlyeq_E \in D(\succcurlyeq)} SE(\succcurlyeq_E)\ \forall \succcurlyeq \in D$.
- A rule f satisfies PO if $f(\succcurlyeq) \subset PO(\succcurlyeq)\ \forall \succcurlyeq \in D$

- A rule f satisfies IMR if $f(\succcurlyeq) \subset \bigcup_{\succcurlyeq_E \in D(\succcurlyeq)} IMR(\succcurlyeq_E)\ \forall \succcurlyeq \in D$.
- A rule f satisfies IR if $f(\succcurlyeq) \subset IR(\succcurlyeq)\ \forall \succcurlyeq \in D$.

- A rule f satisfies LI if
  $\forall \succsim, \succsim' \in D$, $\forall\ z \in Z \cap R^{nl}_{++}$,
  $p(\succsim_i, z_i) = p(\succsim'_i, z_i)\ \forall\ i$,
  then $z \in f(\succsim) \Longleftrightarrow z \in f(\succsim')$.
- $p(\succsim_i, z_i)$: the supporting price of $\succsim_i$ at z_i.

- $\succsim_E$ is essentially identical to $\succsim'_E$ around $z \in R_+^{nl}$ if there exist functions u_i, $\varepsilon_i : R_+^l \longrightarrow R$ s. t. $\forall\ i, j \in N\ \forall x, y \in R_+^l$, $(x, i) \succsim_E (y, j) \Longleftrightarrow u_i(x) \geq u_j(y)$, $(x, i) \succsim'_E (y, j) \Longleftrightarrow u_i(x) + \varepsilon_i(x) \geq u_j(y) + \varepsilon_j(y)$, where $\varepsilon_i(z_i) = 0$ and $\frac{\varepsilon_i(x)}{||x - z_i||} \longrightarrow 0$ as $x_i \longrightarrow z_i$, and the same property holds for ε_j.

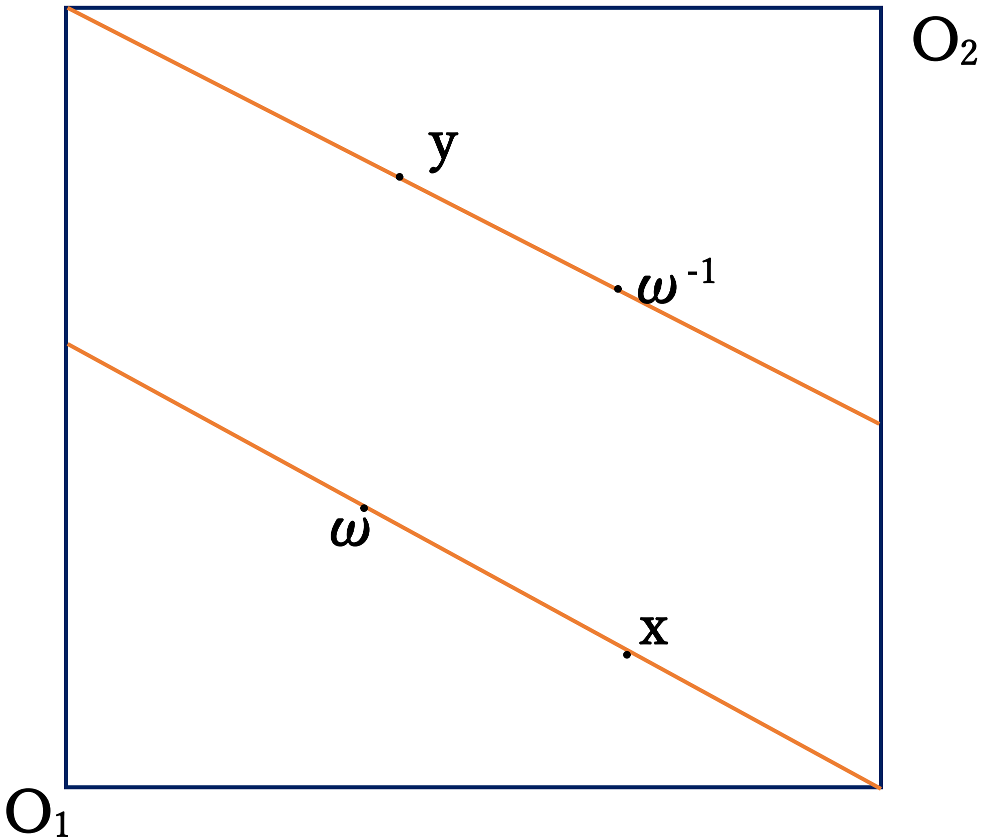
- A rule f satisfies ELI if $\forall \succsim \in D$, $\forall$ $z \in Z \cap R^{nl}_{++}$, if there exists $\exists \succsim_E \in D(\succsim)$, $\exists p \in int.\Delta^l$ s. t. $\succsim_E$ is essentially identical to $\succsim^p_E$ around z, then $z \in f(\succsim) \iff z \in f(\succsim^p)$.
- ELI is equivalent to LI if PO is satisfied (Lemma 5).


i
j

30 Results

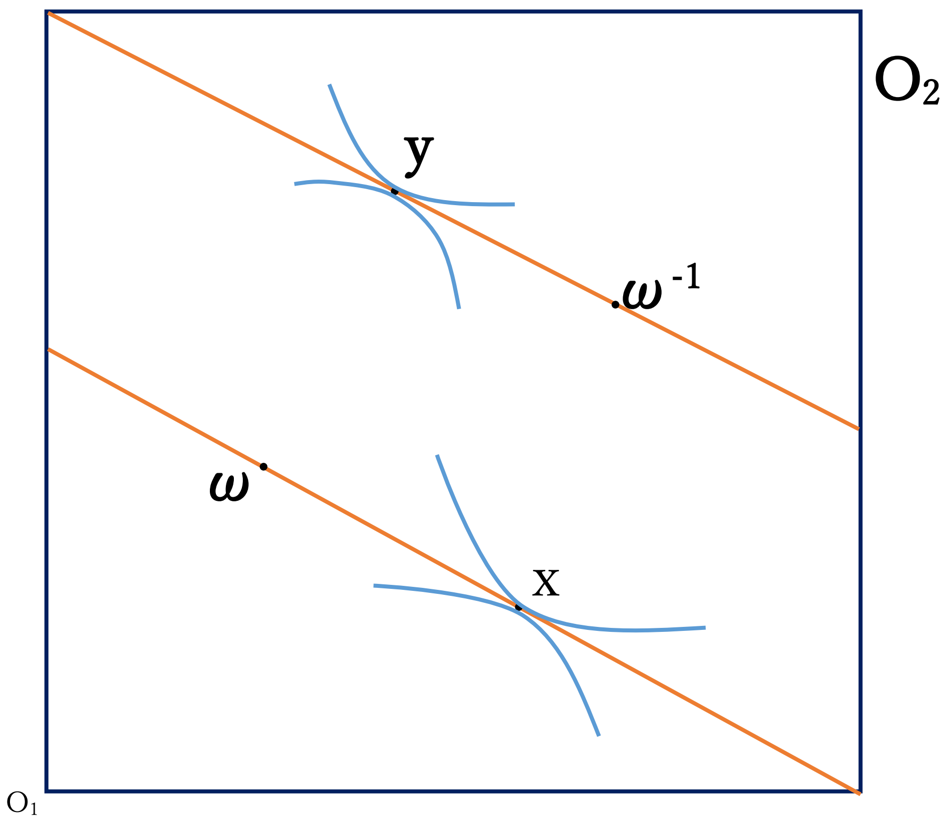
Theorem 1

Assume D.1, D.2, and D.3. The impartial Walras rule is the unique rule satisfying SN, IMR, SE, and ELI.



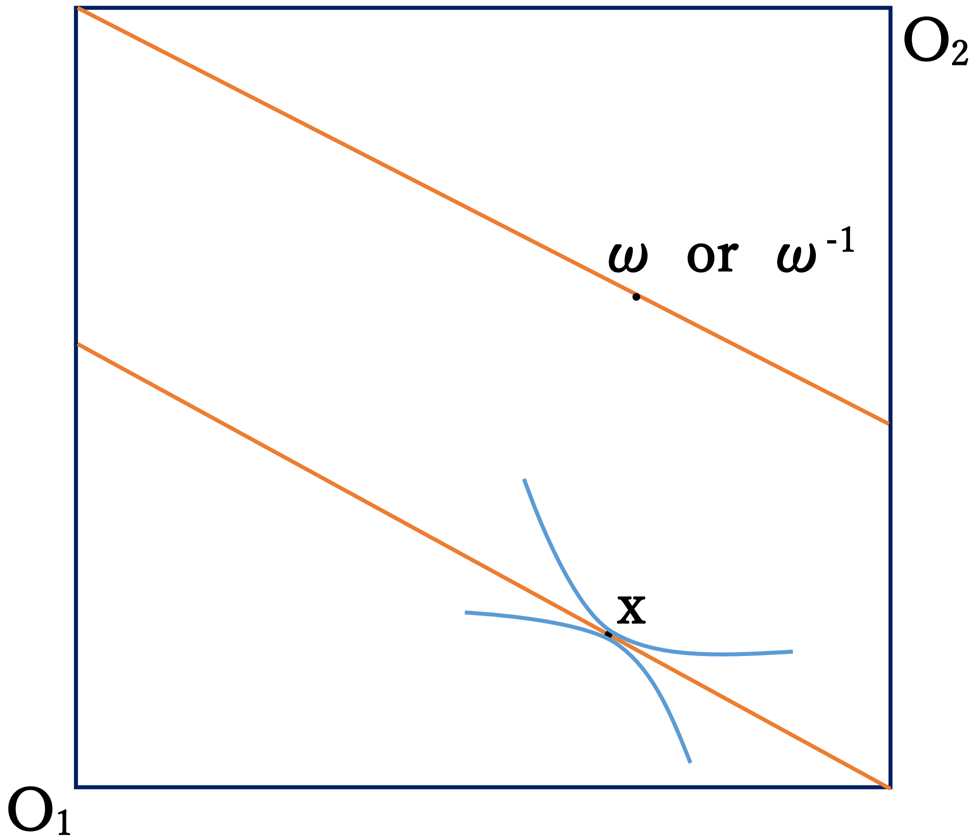
If every has the same linear preference r, the image of rules satisfying

SN and IMR coincide with the red lines.



IW⊂f: x and y are impartial Walrasian allocations.

Lemma shows x∈f(≿). ELI implies x∈f(≿). Thus IW⊂f.



f⊂IW: Suppose x∈f(≿) and x∉IW(≿) ELI implies x∈f(≿), a contradiction.

31 Interpretation 1.”fictional” social choice

A social choice problem where no one knows who has which initial endowment.
Rawls’s veil of ignorance (Rawls 1971).

32 Interpretation 2.”fictional” social choice

A social choice problem in which everyone expects and/or imagines to own other agent’s initial endowment with equal probability. Putting oneself in another’s shoes of Hare (1981) and Harsanyi (1955).

33 Axiomatization of the equal income Walras rule

It is semantically the same as the situation where everyone has the same initial endowment. In this case, the impartial Walras rule is equivalent to the equal income Walras rule studied by Thomson (1988), Nagahisa and Suh (1995), Maniquiet (1996), and Toda (2004).

参考文献

[1] K. J. Arrow, ”Social Choice and Individual Values,” New York: John Wiley, Second Edition,1963.

[2] Blackorby C, Bossert W, Donaldson D (2002) Utilitarianism and the theory of justice. In: Arrow KJ, Sen AK, Suzumura K (eds) handbook of social choice and welfare. Elsevier, Amsterdam, pp 543–596

[3] Blackorby C, Donaldson D, Weymark JA

(1984) Social choice with interpersonal utility comparisons: a diagrammatic introduction. Int Econ Rev 25:327–356

[4] Bossert W, Weymark JA (2004) Utility in social choice. In:Barber, Hammond P, Seidl C (eds) Handbook of utility theory, vol 2. Kluwer, Dordrecht, pp 1099–1177

[5] C.P. Chambers and T. Hayashi (2017), Resource allocation with partial responsibilities for initial endowments, Int. J. Economic Theory, Vol.13, pp355-368.

[6] d'Aspremont C (1985) Axioms for Social welfare orderings.In:Hurwicz L,Schmeidler D,Sonnenschein H (eds) Social goals and social organization: essays in memory of Elisha Pazner. Cambridge University Press, Cambridge, pp 19–67

[7] d'Aspremont C and Gevers L (1977) Equity and informational basis of collective choice, Review of Economic Studies Vol.44, pp199-209

[8] d'Aspremont C and L. Gevers (2002) Social

welfare functionals and interpersonal comparability in Handbook of Social Choice and Welfare Vol.1 Chapter 10 459-541.

[9] Deschamps R, Gevers L (1978) Leximin and utilitarian rules: a joint characterization. J. Econ. Theory 17:143–163

[10] Fleurbaey M, Hammond PJ (2004) Interpersonally comparable utility. In: Barber S, Hammond P, Seidl C (eds) Handbook of utility theory, vol 2. Kluwer, Dordrecht, pp 1179–1285

[11] M. Fleurbaey, K. Suzumura, and K. Tadenuma, (2005) The informational basis of the theory of fair allocation. Social Choice and Welfare 24, pp.311–341.

[12] Gevers L (1979) On interpersonal comparability and social welfare orderings. Econometrica 44:75–90

[13] Gevers L (1986) Walrasian social choice : some simple axiomatic approaches. In:Heller et al.(eds) Social choice and public decision making: essays in honor of Arrow JK Vol 1.

Cambridge Univ. Press, Cambridge
[14] Hammond JP (1976) Equity, Arrow's conditions, and Rawls' difference principle, Econometrica Vol.44 pp793-804
[15] J.P. Hammond (2010), Competitive market mechanisms as social choice procedures, in "Handbook of Social Choice and Welfare Vol.2," K. J. Arrow, A. Sen, and Kotaro Suzumura Eds.), pp 47-152, Amsterdam: Elsevier.
[16] Hammond JP (1991) Interpersonal

comparisons of utility : Why and how they are and should be made, in "Interpersonal Comparisons of Well-Being" (Elster J and JE Roemer Eds.) pp200-254. Cambridge Univ. Press

[17] Hare, R. M. (1981). Moral thinking: its levels, method, and point. Oxford University Press.

[18] Harsanyi, J. C. (1955) "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility," Journal of Political

Economy 63: 309-321

[19] L. Hurwicz, On allocations attainable through Nash equilibria. J. Econ. Theory 21, (1979), pp.140-165.

[20] Maniquet F (1996) Horizontal equity and stability when the number of agents is variable in the fair division problem Economics Letters Vol. 50: 85-90

[21] K. Miyagishima, (2015) Implementability and equity in production economies with unequal skills, Rev. Econ. Design 19,

pp.247–257.
[22] R. Nagahisa, (1991) A local independence condition for characterization of Walrasian allocations rule, J. Econ. Theory 54, pp.106-123.
[23] R. Nagahisa and SC. Suh, (1995) A characterization of the Walras rule, Soc. Choice Welfare 12, pp.335-352; reprented in "The Legacy of Léon Walras Vol.2: Intellectual Legacies in Modern Economics 7" (D. A. Walker, Ed.), pp. 571-588,

Cheltenham (UK): Edward Elgar Publishing Ltd, 2001.

[24] Rawls J (1971) A Theory of Justice Harvard University Press

[25] Roberts K (2010) Social choice theory and the informational basis approach. In: Morris CW, Sen A (eds) Cambridge University Press, Cambridge, pp 115–137

[26] Sakai, T (2009) Walrasian social orderings in exchange economies, J. Math. Econ. 45, pp.16–22.

[27] Sen AK (1970) Collective Choice and Social Welfare Holden-Day:San Francisco.
[28] Sen AK (1974a) Rawls versus Bentham: An axiomatic examination of the pure distridution problem, Theory and Decision 4 301-309.
[29] Sen AK (1974b) Informational bases of alternative welfare approaches: Aggregation and income distribution, Journal of Public Economics 3 387-403.
[30] Sen AK (1976) Welfare inequalities and

Rawlsian axiomatics. Theory Dec 7: 243–262

[31] Sen AK (1977) On weights and measures: Informational constraints in social welfare analysis, Econometrica 45 pp1539-72

[32] Sen AK (1986) Social choice theory. In: Arrow KJ, Intriligator MD (eds) Handbook of mathematical economics, vol 3. North-Holland, Amsterdam, pp 1073–1181

[33] Strasnick S (1976) Social choice and the deviation of Rawls' difference principle,

Journal of Philosophy 73 pp184-194
[34] Suppes P (1966) Some formal models of grading principles. Synthese 6: 284-306
[35] Suzumura K (1983) Rational Choice, Collective Decisions, and Social Welfare, Cambridge Univ. Press.
[36] W. Thomson, (1988) A study of choice correspondences in economies with a variable number of agents, J. Econ. Theory 46, pp.237-254.
[37] M. Toda, (2004) Characterizations of the

Walrasian solution from equal division, mimeo.

[38] K. Urai and H. Murakami, (2015) Local Independence, monotonicity and axiomatic characterization of price-money message mechanism, mimeo.

[39] H. Yamamura, (2017) Interpersonal comparison necessary for Arrovian aggregation, Social Choice and Welfare 49:37–64

[40] N. Yoshihara, (1998) Characterizations of

public and private ownership solutions,
Math. Soc. Science 35, pp.165–184.