The resource allocation problems with interpersonal comparisons of welfare: An axiomatization of the impartial Walras rule

Ryo-Ichi Nagahisa
Department of Economics, Kansai University
3-3-35 Yamatecho Suita 564-8680 Japan

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Abstract

The extended sympathy approach, which has been studied so far in the abstract framework of social choice, is applied to the resource allocation problem of exchange economies with a finite number of agents and a finite number of private goods. The central issue in this paper is the axiomatic analysis of what we call the impartial Walras rule: It associates each (subjective) preference profile with the set of impartial Walras allocations, which are defined as Walrasian allocations operated from permuted initial endowments. We show that the impartial Walras rule is the unique rule that satisfies Suppes nondiscrimination, impartial rationality, Suppes equity, and extended local independence.

1 Introduction

The notion of extended preference makes it possible to compare the welfare of different individuals in social choice. It has been developed mainly in the literature of Arrow's impossibility theorem, in the abstract framework of social choice[1]. The purpose of this paper is applying this approach to the resource allocation problem of exchange economies with a finite number of agents and with a finite number of private goods.

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[1] d'Aspremont (1985), d'Aspremont and Gevers (2002), Sen (1970,1977,1986), and Suzumura (1983) surveyed this area. Blackorby et al. (1984) give a diagrammatic introduction. The most recent contribution is Yamamura (2017).

The exceptions are Sen (1974a,b) and Deschamps and Gevers (1978), which consider the income distribution problem in a single commodity economy.

The central issue of the paper is an axiomatic analysis of what we call the impartial Walras rule, which associates with each (subjective) preference profile the set of impartial Walrasian allocations, which are defined as Walrasian allocations operated from permuted initial endowments. Theorem 1 is the main result, which says that the impartial Walras rule is the unique rule satisfying Suppes nondiscrimination (SN), impartial rationality (IMR), Suppes equity (SE), and extended local independence (ELI). SN is a generalization of anonymity or nondiscrimination and says that any allocations which are supposed to be identical through permuting agents should be dealt with equally in social choice. IMR is a generalization of individual rationality and gives each agent a utility level that is better than the utility attained through a permutation of agents. SE is an interpersonal extension of Pareto optimality. ELI, a generalization of local independence due to Nagahisa (1991), is a requirement of informational economization for interpersonal comparisons of welfare. It works over extended preferences in a similar way as local independence does over subjective preferences. Theorem 1 is the counterpart of the main result of Nagahisa (1991), an axiomatization of the Walras rule[2]: If interpersonal comparisons of welfare are allowed, the similar axioms as those in Nagahisa (1991) lead us to the impartial Walras rule, not to the Walras rule.

We describe the impartial Walras rule as the rule chosen in a fictional social choice arena. An example of the arena is an original position assumed by Rawls (1971) that is covered with the veil of ignorance. A state assumed by Hare (1981) and Harsanyi (1955) where each agent replaces the other's position equally is also another example. We also discuss that if each agent has the identical endowment, Theorem 1 gives an alternative axiomatization of the equal income Walras rule studied by Thomson (1988), Nagahisa and Suh (1995), Maniquet

[2] The study of the axiomatic analysis of the Warlas rule with a finite number of agents is initiated by Gevers (1986) and Hurwicz (1979). Hammond (2002) is a comprehensive survey of the field.

(1996), and Toda (2004).

The paper is organized as follows. Section 2 gives notation and definitions, where extended preferences and Suppes criterion are introduced. The main result and the proof are given in Sections 3 and 4 respectively. Section 5 is the conclusion.

2 Notation and Definitions

2.1 Exchange Economies

We consider exchange economies with a finite number of agents and a finite number of private goods. Let $N = \{1, 2, ..., n\}$ and $L = \{1, 2, ..., l\}$ be the set of the agents and the set of the private goods respectively. All the agents have the same consumption set R^l_+. Let $z_i = (z_{i1}, ..., z_{il}) \in R^l_+$ and $z = (z_1, ..., z_n) \in R^{nl}_+$ be agent i's consumption and an allocation respectively. Let $\omega_i \in R^l_{++}$ be agent i's initial endowment and fixed throughout the paper. Let $\omega = (\omega_1, ..., \omega_n)$. An allocation z is feasible if $\sum\limits_{i \in N} z_i = \sum\limits_{i \in N} \omega_i$. Let Z be the set of feasible allocations.

Let $\succcurlyeq_i$ be agent i's preference on R^l_+. A profile $\succcurlyeq = (\succcurlyeq_i)_{i \in N}$ is a list of all agents' preferences. We consider two types of the set of profiles. Let $\Delta^l := \{p \in R^l_+ : \sum\limits_{i=1}^{l} p_i = 1\}$ and $int.\Delta^l := \{p \in R^l_{++} : \sum\limits_{i=1}^{l} p_i = 1\}$. Let L be the set of profiles such that $\succcurlyeq = (\succcurlyeq_i)_{i \in N} \in L$ if and only if there exists some $p \in int.\Delta^l$ such that for each $\succcurlyeq_i$, $x \succcurlyeq_i y \Longleftrightarrow px \geq py$ for all $x, y \in R^l_+$. That is, a profile $\succcurlyeq$ is in L if and only if every agent has the same preference represented by a linear utility function. We use the notation $\succcurlyeq^p = (\succcurlyeq^p_i)_{i \in N}$ if we need to specify p. Let Q be the set of preferences satisfying (i)-(iii). (i) $\succcurlyeq_i$ is continuous and convex on R^l_+ and continuously differentiable on R^l_{++}. (ii) $x \geq y \& x \neq y$ implies $x \succcurlyeq_i y$, and if in addition $x \in R^l_{++}$, this implies $x \succ_i y$ (monotonicity). (iii) for any $x \in R^l_{++}$, $\{y \in R^l_+ : y \succcurlyeq_i x\} \subset R^l_{++}$ (boundary condition). Let $Q^n = \overbrace{Q \times \cdots \times Q}^{n}$ be the second type of the set of profiles. Let $D = L \cup Q^n$ be called the domain. The domain is the same as that of Nagahisa (1991)[3].

Let a profile $\succcurlyeq \in D$ be given. The Pareto optimal, individually rational, and Walrasian allocations for the profile are defined as usual: (i) $z \in Z$ is Pareto optimal if and only if there is no feasible allocation z' such that $z'_i \succcurlyeq_i z_i$ for all $i \in N$ and $z'_i \succ_i z_i$ for some $i \in N$: (ii) z is individually rational if and only if $z_i \succcurlyeq_i \omega_i$ for all $i \in N$:(iii) z is a Walrasian allocation if and only if there is a price vector $p \in int.\Delta^l$ such that for all $i \in N$, $z_i \succcurlyeq_i x_i$ for all $x_i \in R^l_+$ such that $px_i \leq p\omega_i$. Let $PO(\succcurlyeq)$, $IR(\succcurlyeq)$, and $W(\succcurlyeq)$ stand for the set of Pareto optimal, individually rational, and Walrasian allocations for $\succcurlyeq$ respectively. We occasionally use the notation $W(\succcurlyeq, \omega)$ and $IR(\succcurlyeq, \omega)$ instead of $W(\succcurlyeq)$ and $IR(\succcurlyeq)$.

2.2 Extended Preferences

The notion of extended preferences is based on the principle of the extended sympathy, mentioned by Arrow (1963) and initiated by Suppes (1966) and Sen (1970). The basic idea is that a hypothetically existing ethical observer compares the welfare of different persons from a social point of view while respecting (or sympathizing with) their subjective preferences. Given a profile $\succcurlyeq \in D$, an extended preference $\succcurlyeq_E$ generated from $\succcurlyeq$ is a complete and transitive binary relation on $R^l_+ \times N$. We read $(x, i) \succcurlyeq_E (y, j)$ as "being agent i with consumption x is at least as well off as being agent j with consumption y".[4] We assume that for any $i \in N$, the restriction of $\succcurlyeq_E$ to $R^l_+ \times \{i\}$ is identical to $\succcurlyeq_i$; $x \succcurlyeq_i y \iff (x, i) \succcurlyeq_E (y, i)$ for any $x, y \in R^l_+$ and any $i \in N$. This assumption has been referred to in the literature as the axiom of identity. It requires that individual preferences be respected in extended preferences. This axiom

[3] The domain Nagahisa and Suh (1995) employed is more natural. But we prefer mathematical tractability here.

[4] Note that we admit interpersonal welfare comparisons, but still deny cardinality.

constitutes the core idea of the sympathy of the ethical observer.[5]

The examples below illustrate two extended preferences, both of which play important roles in the subsequent sections.

Example 1 Let $\succcurlyeq \in D$ and $p \in int.\Delta^l$ be given. The extended preference $\succcurlyeq_{E(p)}$ is defined by

$$(x,i) \succcurlyeq_{E(p)} (y,j) \Longleftrightarrow \min\{pq : q \sim_i x\} \geq \min\{pq : q \sim_i y\}^6.$$

Let p be prices. In $\succcurlyeq_{E(p)}$, the price p is used as a common indicator of interpersonal comparisons of welfare: Being agent i with consumption x is at least as well off as being agent j with consumption y if and only if the minimum amount of expense that agent i needs to spend to achieve the level of utility at x is not smaller than that at agent j's y. If $\succcurlyeq^p \in L$, then $\succcurlyeq^p_{E(p)}$ reduces to a simple form as follows.

Example 2 $(x,i) \succcurlyeq^p_{E(p)} (y,j) \Longleftrightarrow px \geq py.$

We can identify subjective preference itself with the extended preference. To simplify the notation, we denote this extended preference by $\succcurlyeq^p_E$ hereafter.

2.3 Suppes criterion

Given an allocation z and a permutation π on N, we let z^π be an allocation such that $z^\pi_i = z_{\pi(i)}$ for every $i \in N$. Let Π be the set of permutations. We introduce the Suppes criterion (Suppes 1966), occasionally called the grading principle and interpreted as an interpersonal extension of the Pareto criterion. Let $\succcurlyeq_E$ and allocations z, z' be given: According to the criterion, (i) z is at least as just as z' for $\succcurlyeq_E$ if and only if there is some $\pi \in \Pi$ such that (z_i, i) $\succcurlyeq_E$ $(z'_{\pi(i)}, \pi(i))$ for all $i \in N$: (ii) z is more just than z' for $\succcurlyeq_E$ if and only if (i) holds with strict preference relation $(z_i, i) \succ_E (z'_{\pi(i)}, \pi(i))$ for at least one

[5] This is implicitly assumed in almost every literature related to extended preferences. Refer to Sen (1970) and d'Aspremont (1985) for more details.

[6] Note that $\min\{pq : q \sim_i x\} = 0$ if $\succcurlyeq_i \in Q$ and $x \notin R^l_{++}$.

member i : (iii) z is equally as just as z' for $\succsim_E$ if and only if there is some $\pi \in \Pi$ such that $(z_i, i) \sim_E (z'_{\pi(i)}, \pi(i))$ for all $i \in N$. It is obvious that all of the three relations are well defined and transitive, and that "equally as just as" relation is symmetric whereas "more just than" relation is asymmetric. The Suppes criterion is used in the axiomatization of the utilitarian rule and the leximin rule in the literature of Arrovian social choice[7].

Let $\succsim_E$ be given. A feasible allocation z is Suppes equitable for $\succsim_E$ if and only if there are no feasible allocations z' that are more just than z for $\succsim_E$. Let $SE(\succsim_E)$ be the set of Suppes equitable allocations. Let $IMR(\succsim_E)$ be the set of feasible allocations that are at least as just as ω, called the set of impartial rational allocations. The axiom of identity implies that $SE(\succsim_E) \subset PO(\succsim)$ and $IR(\succsim) \subset IMR(\succsim_E)$ for all $\succsim_E$. Note that $SE(\succsim_E)$ is nonempty if there exist utility functions u_i $(i \in N)$ such that $(x, i) \succsim_E (y, j) \Longleftrightarrow u_i(x) \geq u_j(y)$ for all $i, j \in N$ and $x, y \in R^l_+$: Feasible allocations $z = (z_i)_{i \in N}$ maximizing $\sum_{i \in N} u_i(z_i)$ on Z are Suppes equitable. This assures that $SE(\succsim_{E(p)})$ is nonempty.

2.4 Rules

Let $f : D \longrightarrow Z$ be a social choice rule, simply a rule hereafter, which associates with each profile a nonempty subset of feasible allocations. A rule f decides $f(\succsim)$ through interpersonal comparisons of welfare. In other words, it uses extended preferences generated from $\succsim$ to decide $f(\succsim)$. But not all the extended preferences are used.

Given a profile $\succsim \in D$, let $D(\succsim)$ be a nonempty subset of $\succsim_E$. A rule f decides $f(\succsim)$ based on extended preferences in $D(\succsim)$, and no extended preferences excluded from $D(\succsim)$ are used in this decision. Let us call $\underset{\succsim \in D}{\cup} D(\succsim)$ the extended domain. We consider the following three assumptions on the extended

[7] The contributions are Sen (1976), d'Aspremont and Gevers (1977), Deschamps and Gevers (1978), Gevers (1979), Hammond (1976,79), and Strasnick (1976).

domain.

D.1. For any $\succcurlyeq^p \in L$, then $D(\succcurlyeq^p) = \{\succcurlyeq^p_E\}$.

D.2. For any $\succcurlyeq \in D$ and any $\succcurlyeq_E \in D(\succcurlyeq)$, $(x,i) \succcurlyeq_E (0,j)$ for all $i,j \in N$ and all $x \in R^l_+$.

D.3. For any $\succcurlyeq \in D$ and any $p \in int.\Delta^l$, $\succcurlyeq_{E(p)} \in D(\succcurlyeq)$.

There exist many extended domains satisfying the three conditions. The smallest one is $D(\succcurlyeq) = \{\succcurlyeq_{E(p)}: p \in int.\Delta^l\}$ for any $\succcurlyeq \in D$. The largest one is defined by comparison of utilities among agents subject to $u_i(0)$ being equal across all i.

There is no compelling reason to dismiss D.1: If everyone has the same linear preference, that preference needs to be the extended preference and no other extended preferences are considered possible. It is very hard to reject D.2, which reflects our intuition that as long as other conditions are equal, people without wealth are the most miserable in the world. D.3 can be justified as follows. Let p be the supporting price at z. If we look only around z, we can think of $\succcurlyeq_{E(p)}$ as being identical to $\succcurlyeq^p$: They are locally identical around z. To create extended preferences, let us request that only the local information about preferences in this sense be used. Then, as long as $\succcurlyeq^p$ is allowed as an extended preference, so is $\succcurlyeq_{E(p)}$. This is the legitimacy of D.3.

The Walras rule W associates with each profile the set of Walrasian allocations for the profile. The impartial Walras rule is a generalization of the Walras rule, which is defined as follows. Let $\succcurlyeq \in D$ and $\pi \in \Pi$ be given. Let $W(\succcurlyeq, \omega^\pi)$ be a set such that $z \in W(\succcurlyeq, \omega^\pi)$ if and only if z is a Walrasian allocation when all agents' endowments are permuted through π: Given a price vector $p \in int.\Delta^l$, every agent i chooses z_i as his best choice when his endowment is $\pi(i)$'s one. Let $\underset{\pi\in\Pi}{\cup} W(\succcurlyeq, \omega^\pi)$ be called the set of impartial Walrasian allocations for $\succcurlyeq$. The impartial Walras rule $IW : D \longrightarrow Z$ is a rule that associates with each $\succcurlyeq \in D$ the set of impartial Walrasian allocations for the profile. The equal income Walras rule is the Walras rule when every agent has the same initial endowment. The impartial Walras rule reduces to the equal income Walras rule in the case.

2.5 Axioms

A rule f satisfies **Suppes Nondiscrimination** (**SN**) if for any $\succcurlyeq \in D$, and for any $z, z' \in Z$, z is equally as just as z' for any $\succcurlyeq_E \in D(\succcurlyeq)$, then $z \in f(\succcurlyeq) \Longleftrightarrow z' \in f(\succcurlyeq)$. A rule f satisfies **Suppes Equity** (**SE**) if $f(\succcurlyeq) \subset \bigcup\limits_{\succcurlyeq_E \in D(\succcurlyeq)} SE(\succcurlyeq_E)$ for any $\succcurlyeq \in D$. A rule f satisfies **Impartial Rationality** (**IMR**) if $f(\succcurlyeq) \subset \bigcup\limits_{\succcurlyeq_E \in D(\succcurlyeq)} IMR(\succcurlyeq_E)$ for any $\succcurlyeq \in D$.

SE is well defined: Take $z \in PO(\succcurlyeq)$ arbitrarily. Let $p \in int.\Delta^l$ be the supporting price at z. It is easy to see $z \in SE(\succcurlyeq_{E(p)})$, and hence $z \in SE(\succcurlyeq_{E(p)}) \overset{D.3}{\overset{\downarrow}{\subset}} \bigcup\limits_{\succcurlyeq_E \in D(\succcurlyeq)} SE(\succcurlyeq_E) \neq \emptyset$.

Suppes Equity and Impartial Rationality correspond to Pareto Optimality and Individual Rationality of the framework of subjective preferences respectively. Just as Pareto Optimality is replaced by Suppes Equity, Individual Rationality must also be replaced by Impartial Rationality. Note that the strength of each axiom depends on the size of $D(\succcurlyeq)$. The richer the $D(\succcurlyeq)$, the weaker the axiom.

Let $\succcurlyeq \in D$ and $z_i \in R^l_{++}$ be given. We say that $p \in int.\Delta^l$ is the supporting price of $\succcurlyeq$ at z_i if agent i's indifference curve passing through z_i is tangent to the line $\{x : px = pz_i\}$ at z_i. Let $z \in R^{nl}_{++}$ be given. We say that $p \in int.\Delta^l$ is the supporting price of $\succcurlyeq$ at z if for any i, $p \in int.\Delta^l$ is the supporting price of $\succcurlyeq$ at z_i.

Let $\succcurlyeq = (\succcurlyeq_i)_{i \in N} \in D$ and $z = (z_i)_{i \in N} \in R^{nl}_{++}$ be given. Let $p(\succcurlyeq_i, z_i)$ be the supporting price of $\succcurlyeq_i$ at z_i.

A rule f satisfies Local Independence (LI) if for any $\succcurlyeq, \succcurlyeq' \in D$ and any $z \in Z \cap R^{nl}_{++}$, if $p(\succcurlyeq_i, z_i) = p(\succcurlyeq'_i, z_i)$ for all i, then $z \in f(\succcurlyeq) \Longleftrightarrow z \in f(\succcurlyeq')$.

LI was used in the axiomatization of the Walras rule by Nagahisa (1991).[8] We can show that LI together with the other axioms characterizes the impartial Walras rule. However, LI is a requirement defined in subjective preferences, and hence we must seek a more sophisticated version defined in extended preferences. The idea behind LI is that only the local information around z is used for social choice. Just as LI applies this idea to individual preferences, the extended local independence, a generalization of LI, applies it to extended preferences. Roughly speaking, it says that if two extended preferences with totally different forms as a whole are considered as identical around z, they have the same implication for social choice. The formal definition is as follows.

Let $\succsim, \succsim' \in D$ and $z \in R_+^{nl}$ be given. Let $\succsim_E$ and $\succsim'_E$ be extended preferences generated from $\succsim$ and $\succsim'$ respectively. We say that $\succsim_E$ is essentially identical to $\succsim'_E$ around z if there exist functions u_i, $\varepsilon_i : R_+^l \longrightarrow R$ such that for all $i, j \in N$ and all $x, y \in R_+^l$,

$(x, i) \succsim_E (y, j) \Longleftrightarrow u_i(x) \geq u_j(y)$,

$(x, i) \succsim'_E (y, j) \Longleftrightarrow u_i(x) + \varepsilon_i(x) \geq u_j(y) + \varepsilon_j(y)$,

where $\varepsilon_i(z_i) = 0$ and $\frac{\varepsilon_i(x)}{||x - z_i||} \longrightarrow 0$ as $x_i \longrightarrow z_i$, and the same property holds for ε_j.

We regard u_i and ε_i as a utility function representing $\succsim_i$ and a higher order error term respectively. We regard u_j and ε_j as well.

A rule f satisfies **Extended Local Independence** (**ELI**) if for any $\succsim \in D$ and any $z \in Z \cap R_{++}^{nl}$, if there exists some $\succsim_E \in D(\succsim)$ and some $p \in int.\Delta^l$ such that $\succsim_E$ is essentially identical to $\succsim_E^p$ around z, then $z \in f(\succsim) \Longleftrightarrow z \in f(\succsim^p)$.

Suppose that $p \in int.\Delta^l$ is the supporting price of $\succsim \in D$ at $z \in Z \cap R_{++}^{nl}$. If extended preferences that are used to decide whether $z \in f(\succsim)$ or not should be

[8] Refer to Nagahisa (1991) for more details on LI. Generalizations and related axioms of LI are found in Yoshihara (1998), Fleurbaey, et.al (2005), and Miyagishima (2015). An ordering version of LI is given by Sakai (2009). Urai and Murakami (2015) uses LI for economies with money.

created from the information of individual preferences only around z, it must be essentially identical to $\succsim_E^p$ around z. Such the extended preferences are used for deciding $z \in f(\succsim)$ or not[9], not all extended preferences in $D(\succsim)$ are used: Extended preferences that are not essentially identical to $\succsim_E^p$ around z are not considered. As D.1 requires that $\succsim_E^p$ be the only extended preferences in $D(\succsim^p)$, this means that deciding whether $z \in f(\succsim)$ or not is equivalent to deciding whether $z \in f(\succsim^p)$ or not. This is the essence of ELI.

We show that ELI is equivalent to LI if PO is satisfied (Lemma 5). ELI says nothing in the case of no supporting price. However, this is enough for the axiomatization of the impartial Walras rule. No further discussion is necessary.

3 Results

Theorem 1 *Assume D.1, D.2, and D.3. The impartial Walras rule is the unique rule satisfying SN, IMR, SE, and ELI.*

The proof of Theorem 1 follows along the similar line as that of the axiomatization of the Walras rule due to Nagahisa (1991). As Suppes nondiscrimination, Suppes equity, and extended local independence are essentially equivalent to nondiscrimination, Pareto optimality, and local independence in Nagahisa respectively, we conclude that impartial rationality is the most responsible for the axiomatization of the Impartial Walras rule.

It is believed that the impartial Walras rule is chosen in a fictional social choice. Two interpretations of "fictional" are open to us. One interpretation is to think of it as a social choice problem where no one knows who has which initial endowment.[10] This interpretation reminds us of Rawls's veil of ignorance

[9] The existence of such extended preferences is assured by Lemma 3 in Section 4, which shows that $\succsim_{E(p)}$ is one of the extended preferences.

[10] Speaking more accurately, we consider a problem where everyone knows the set of initial endowments, but no one knows who owns which of them.

(Rawls 1971). The similar logic as Rawls works with this interpretation. The impartial Walras rule probably may not be supported if the veil of ignorance was removed, i.e., if everyone knew their endowments perfectly. In that state, the Walras rule will be more favored. But otherwise, the impartial Walras rule could be justified.

A social choice problem in which everyone expects and/or imagines to own other agent's initial endowment with equal probability can also be the other interpretation. This idea, putting oneself in another's shoes, is reminiscent of the suppositions of Hare (1981) and Harsanyi (1955). Moreover, it is semantically the same as the situation where everyone has the same initial endowment, i.e., everyone owns $\frac{1}{n}\sum\limits_{i\in N}\omega_i$ initially. In this case, the impartial Walras rule is equivalent to the equal income Walras rule studied by Thomson (1988), Nagahisa and Suh (1995), Maniquet (1996), and Toda (2004), and hence Theorem 1 can be regarded as an axiomatization of the equal income Walras rule in the case.

The four examples below illustrate that the axioms are independent and that Theorem 1 does not hold if one of the axioms lacks.

Example 3 (The impartial Walras rule operated from different endowments) Let ϖ be a new initial endowment such that $\varpi_i = \frac{\omega_i}{2}$ $(i = 1, ..., n-1)$ and $\varpi_n = \omega_n + \frac{1}{2}\sum\limits_{i=1}^{n-1}\omega_i$. The rule f is given by $f(\succcurlyeq) = \bigcup\limits_{\pi\in\Pi} IW(\succcurlyeq, \varpi^{\pi})$ for any $\succcurlyeq \in D$. This rule satisfies all the axioms except for IMR.

Example 4 (The Walras rule) The Walras rule satisfies all the axioms except for SN.

Example 5 (The impartial Walras plus Ω rule) Let $\Omega(\succcurlyeq) = \bigcup\limits_{\pi\in\Pi}\left\{z \in Z : z_i \sim_i \omega_{\pi(i)} \forall i\right\}$.

$$f(\succcurlyeq) = \begin{cases} IW(\succcurlyeq) \cup \Omega(\succcurlyeq) & \text{if } \succcurlyeq \in Q^n \text{ and } \succcurlyeq_1 = \cdots = \succcurlyeq_n \\ IW(\succcurlyeq) & \text{otherwise} \end{cases} \quad \text{for any } \succcurlyeq \in D$$

This rule satisfies all the axioms except for SE. We can easily understand that f satisfies IMR if we notice $z \in IMR(\succsim_{E(p)})$ for any $z \in \Omega(\succsim)$.[11] Refer to the next section to see that f satisfies SN and ELI. Note that $\omega \in \Omega(\succsim)$ is not always Pareto optimal. Thus SE is violated.

Example 6 (The impartial core rule) Let $Core(\succsim)$ be the set of core allocations for $\succsim$[12]. Let f be a rule such that $z \in f(\succsim)$ if and only if there exists some $z' \in Core(\succsim)$ that is equally as just as z for any $\succsim_E \in D(\succsim)$. This rule, a generalization of the core rule, is well defined because of $Core(\succsim) \subset f(\succsim)$. This rule satisfies all the axiom except for ELI.

4 Proofs

The proof follows along the same line as Nagahisa (1991). The four axioms, impartial rationality, Suppes nondiscrimination, Suppes equity, and extended local independence play the same role as in individual rationality, nondiscrimination, Pareto optimality, and local independence there respectively.

Lemma 1 *The Impartial Walras rule satisfies SE, SN, and IMR.*

Proof. SE: Let $z \in IW(\succsim)$ and $p \in int.\Delta^l$ be the price associated with z. Take $\succsim_{E(p)} \in D(\succsim)$. We show $z \in SE(\succsim_{E(p)})$ that completes the proof. Suppose not. There exists some $z' \in Z$ such that z' is more just than z for $\succsim_{E(p)}$. This implies that there exists some $\pi \in \Pi$ such that $(z'_i, i) \succsim_{E(p)} (z_{\pi(i)}, \pi(i))$ for all i and $(z'_i, i) \succ_{E(p)} (z_{\pi(i)}, \pi(i))$ for some i. By definition of $\succsim_{E(p)}$, this further implies $pz'_i \geq pz_{\pi(i)}$ for all i and $pz'_i > pz_{\pi(i)}$ for some i. This contradicts the feasibility of z and z'.

[11] For any $z \in \Omega(\succsim)$, $\min\{pq : q \sim_i z_i\} \overset{z\in\Omega(\succsim)}{\overset{\downarrow}{=}} \min\{pq : q \sim_i \omega_{\pi(i)}\} \overset{\succsim_i = \succsim_{\pi(i)}}{\overset{\downarrow}{=}} \min\{pq : q \sim_{\pi(i)} \omega_{\pi(i)}\}$. Thus $(z_i, i) \succsim_{E(p)} (\omega_{\pi(i)}, \pi(i))$.

[12] Core allocations are defined as usual by using weak and strict preference relations.

SN: Let $z \in IW(\succsim)$ and $p \in int.\Delta^l$ be the price associated with z. Let $z' \in Z$ be such that z is equally as just as z' for any $\succsim_E \in D(\succsim)$. By definition of IW, there is some $\pi \in \Pi$ such that for all $i \in N$,

(1) $z_i \succsim_i x_i$ for all $x_i \in R^l_+$ such that $px_i \leq p\omega_{\pi(i)}$.

By noting that $\succsim_{E(p)} \in D(\succsim)$ and that z is equally as just as z' for $\succsim_{E(p)}$, there is some $\rho \in \Pi$ such that

(2) $(z_{\rho(i)}, \rho(i)) \sim_{E(p)} (z'_i, i)$ for all $i \in N$.

Thus we have

(3) $pz_{\rho(i)} = \min\{pq : q \sim_{\rho(i)} z_{\rho(i)}\} = \min\{pq : q \sim_i z'_i\}$.

The first equation of (3) follows from that p is the supporting price of z, and the second from the definition of $\succsim_{E(p)}$and (2). Obviously (3) implies $pz_{\rho(i)} \leq pz'_i$ for all $i \in N$, which, due to the feasibility of z and z', implies

(4) $pz_{\rho(i)} = pz'_i$ for all $i \in N$.

Substituting (4) for (3), we know $pz'_i = \min\{pq : q \sim_i z'_i\}$ and hence

(5) p is the supporting price at z'_i.

On the other hand, (1) implies $pz_i = p\omega_{\pi(i)}$ for all $i \in N$ and hence

(6) $pz_{\rho(i)} = p\omega_{\pi(\rho(i))}$ for all $i \in N$.

(4) and (6) imply

(7) $pz'_i = p\omega_{\pi(\rho(i))}$ for all $i \in N$,

which together with (5) assures that z' is a Walrasian allocation when all agents' endowments are permuted through $\pi \circ \rho$, which completes the proof of SN.

IMR: Let $z \in IW(\succsim)$. By definition of IW, there are some $p \in int.\Delta^l$ and some $\pi \in \Pi$ such that for all $i \in N$, $z_i \succsim_i x_i$ for all $x_i \in R^l_+$ such that $px_i \leq p\omega_{\pi(i)}$. Let $x^*_{\pi(i)}$ be the best on the set $\{x_{\pi(i)} \in R^l_+ : px_{\pi(i)} \leq p\omega_{\pi(i)}\}$ with respect to $\succsim_{\pi(i)}$. Noting that $\succsim_{E(p)} \in D(\succsim)$ and p is the supporting price at z_i and $x^*_{\pi(i)}$, we have $(z_i, i) \sim_{E(p)} (x^*_{\pi(i)}, \pi(i))$. By definition of $x^*_{\pi(i)}$, we have $x^*_{\pi(i)} \succsim_{\pi(i)} \omega_{\pi(i)}$, which together with axiom of identity implies

$(x^*_{\pi(i)}, \pi(i)) \succsim_{E(p)} (\omega_{\pi(i)}, \pi(i))$. Thus we conclude $(z_i, i) \succsim_{E(p)} (\omega_{\pi(i)}, \pi(i))$. This is true for all $i \in N$, and so $z \in IMR(\succsim_{E(p)}, \omega) \subset \bigcup_{\succsim_E \in D(\succsim)} IMR(\succsim_E, \omega)$, which completes the proof of IMR. ■

Let $Z^p = \bigcup_{\pi \in \Pi} \{z \in Z : pz_i = p\omega_{\pi(i)} \ \forall i\}$.

Lemma 2 *Let f a rule satisfying IMR and SN. Then we have $f(\succsim^p) = Z^p$ for any $p \in int.\Delta^l$.*

Proof. Let $z \in f(\succsim^p)$. IMR and D.1 mean that there is some $\pi \in \Pi$ such that $pz_i \geq p\omega_{\pi(i)}$ for all $i \in N$. Due to the feasibility of z, we have $pz_i = p\omega_{\pi(i)}$ for all $i \in N$, which means $z \in Z^p$. Thus we have $f(\succsim^p) \subset Z^p$.

Next, take $z \in Z^p$ arbitrarily. The previous argument assures the existence of $z' \in f(\succsim^p) \subset Z^p$. By definition of Z^p, there exist π and ρ such that $pz_i = p\omega_{\pi(i)}$ and $pz'_i = p\omega_{\rho(i)}$ for all i. Thus $pz'_{\rho^{-1}(i)} = p\omega_i = pz_{\pi^{-1}(i)}$ for all i. By noting D.1, SN and $z' \in f(\succsim^p)$ imply $z \in f(\succsim^p)$. Thus we also have $f(\succsim^p) \supset Z^p$. ■

Lemma 3 *For any $\succsim \in D$ and any $z \in Z \cap R^{nl}_{++}$, the followings are equivalent:*

(i) There exists the supporting price $p \in int.\Delta^l$ of $\succsim$ at z.

(ii) $\succsim_{E(p)}$ is essentially identical to $\succsim^p_E$ around z.

Proof. It is easy to see that (ii) implies (i), just by noting that $u'_i = u_i + \varepsilon_i$ represents $\succsim'_i$. We show the converse. Let u_i represent $\succsim_i$ such that $u_i(x) = \min\{pw : w \sim_i x\}$. The proof goes on with several steps.

Step 1. For each $h \in L$, the marginal utility of h at z_i is equal to p_h; i.e.,$\frac{\partial u_i(z_i)}{\partial x_{ih}} = p_h$.

Proof of Step 1: Take $\varepsilon \neq 0$ arbitrarily. Let $\varepsilon^h = (0, ..., 0, \overset{h}{\varepsilon}, 0, ..., 0) \in R^l$. As $u_i(z_i) = pz_i$ and $u_i(z_i + \varepsilon^h) = \min\{pq : q \sim_i z_i + \varepsilon^h\} \leq p(z_i + \varepsilon^h) = pz_i + p_h\varepsilon$, we have $\frac{u_i(z_i+\varepsilon^h) - u_i(z_i)}{\varepsilon} \leq \frac{pz_i + p_h\varepsilon - pz_i}{\varepsilon} = p_h$. Letting $\varepsilon \to 0$, we have $\frac{\partial u_i(z_i)}{\partial x_{ih}} \leq p_h$. Next, imagine the budget constraint line with the initial endowment of $z_i - \varepsilon^h$

and the price of p.[13] Let z_i^ε be i's optimal consumption under the budget constraint. Obviously, we have $u_i(z_i - \varepsilon^h) \leq u_i(z_i^\varepsilon) = pz_i^\varepsilon = pz_i - p_h\varepsilon$, and hence $\frac{u_i(z_i)-u_i(z_i-\varepsilon^h)}{\varepsilon} = \frac{pz_i-u_i(z_i-\varepsilon^h)}{\varepsilon} \geq \frac{pz_i-(pz_i-p_h\varepsilon)}{\varepsilon} = p_h$. Letting $\varepsilon \to 0$, we have $\frac{\partial u_i(z_i)}{\partial x_{ih}} \geq p_h$, which completes the proof of Step 1.

Step 2. $u_i(x) = px_i + \varepsilon_i(x_i)$, where $\frac{\varepsilon_i(x_i)}{||x_i-z_i||} \longrightarrow 0$ as $x_i \longrightarrow z_i$ and $\varepsilon_i(z_i) = 0$.

Proof of Step 2: Because of the differentiability of u_i, we have

$u_i(x_i) = u_i(z_i) + \sum_{h=1}^{l} \frac{\partial u_i(z_i)}{\partial x_{ih}}(x_{ih} - z_{ih}) + \varepsilon_i(x_i)$, where $\frac{\varepsilon_i(x_i)}{||x_i-z_i||} \longrightarrow 0$ as $x_i \longrightarrow z_i$ and $\varepsilon_i(z_i) = 0$.

As $\frac{\partial u_i(z_i)}{\partial x_{ih}} = p_h$ (Step 1), we have

$u_i(x_i) = u_i(z_i) + \sum_{h=1}^{l} p_h(x_{ih} - z_{ih}) + \varepsilon_i(x_i) = u_i(z_i) + p(x_i - z_i) + \varepsilon_i(x_i)$.

As $u_i(z_i) = pz_i$, we have

$u_i(x_i) = pz_i + p(x_i - z_i) + \varepsilon_i(x_i) = px_i + \varepsilon_i(x_i)$, which completes the proof of Step 2.

Step 3. We complete the proof.

Step 2 shows

$(x, i) \succcurlyeq_{E(p)} (y, j) \Longleftrightarrow u_i(x) \geq u_j(y) \Longleftrightarrow px + \varepsilon_i(x) \geq py + \varepsilon_j(y)$

This means that $\succcurlyeq_{E(p)}$ is essentially identical around z to $\succcurlyeq_E^p$. ■

We say that a rule f satisfies Pareto Optimality (PO) if $f(\succcurlyeq) \subset PO(\succcurlyeq)$ for any $\succcurlyeq \in D$.

Lemma 4 *For any rule f satisfying PO, f satisfies ELI if and only if it satisfies LI.*

Proof. ELI implies LI: Suppose that $p(\succcurlyeq_i, z_i) = p(\succcurlyeq'_i, z_i)$ for any i, where $\succcurlyeq, \succcurlyeq' \in D$ and any $z \in Z \cap R^{nl}_{++}$. Let $z \in f(\succcurlyeq)$. As f satisfies PO, this implies that for some $p \in int.\Delta^l$, $p(\succcurlyeq_i, z_i) = p(\succcurlyeq'_i, z_i) = p$ for any i. Lemma 3 implies that $\succcurlyeq_{E(p)}$ and $\succcurlyeq'_{E(p)}$ are essentially identical to $\succcurlyeq_E^p$ around z. Thus we have

[13] We make ε small enough to satisfy $z_i - \varepsilon^h \gg 0$.

$z \in f(\succsim) \overset{\overset{ELI}{\downarrow}}{\Longrightarrow} z \in f(\succsim^p) \overset{\overset{ELI}{\downarrow}}{\Longrightarrow} z \in f(\succsim')$, which completes $z \in f(\succsim) \Longrightarrow z \in f(\succsim'$). We can prove $z \in f(\succsim) \Longleftarrow z \in f(\succsim')$ similarly.

LI implies ELI: Suppose that there exists some $\succsim_E \in D(\succsim)$ and some $p \in int.\Delta^l$ such that $\succsim_E$ is essentially identical to $\succsim_E^p$ around $z \in Z \cap R_{++}^{nl}$. This implies that p is the supporting price of $\succsim$ at z. Thus LI implies $z \in f(\succsim) \Longleftrightarrow z \in f(\succsim^p)$, the desired result. ■

Proof of Theorem 1. By noting that SE implies PO, Lemma 4 implies that ELI is equivalent to LI. It is easy to see that the impartial Walras rule satisfies LI. Let f be a rule satisfying SN, IMR, SE, and LI. The only remaining thing to prove is $f = IW$.

$IW \subset f$: Take $z \in IW(\succsim)$ arbitrarily. By definition of IW, i weakly prefers z_i to $\omega_{\pi(i)}$. Suppose $z \notin R_{++}^{nl}$. This implies $z_i \notin R_{++}^l$ for some i. If $\succsim \in Q^n$, the boundary condition requires $\omega_{\pi(i)} \succ_i z_i$, which is a contradiction. So we assume $\succsim \in L$. Let p be such that $u_i(x) = px$ represent $\succsim_i$. As i weakly prefers z_i to $\omega_{\pi(i)}$, we have $pz_i \geq p\omega_{\pi(i)}$. As this holds for all i, we have $pz_i = p\omega_{\pi(i)}$ for all i, and hence $z \in Z^p$. Lemma 2 implies $z \in f(\succsim)$, the desired result.

Next, consider the case of $z \in R_{++}^{nl}$. Let p be an equilibrium price associated with z. We have $pz_i = p\omega_{\pi(i)}$ for all i, and hence $z \in Z^p$. Lemma 2 shows $z \in f(\succsim^p)$. As we assumed $z \in R_{++}^{nl}$, LI implies $z \in f(\succsim)$, the desired result.

$f \subset IW$: Take $z \in f(\succsim)$ arbitrarily. Suppose $\succsim \in L$. Lemma 2 implies that there exists some $\pi \in \Pi$ such that $pz_i = p\omega_{\pi(i)}$ for all i, where $p \in int.\Delta^l$ and $\succsim_i$ is represented by $u_i(x) = px$. Thus $z \in IW(\succsim)$, the desired result. Next, consider the case of $\succsim \in Q^n$. Assume $z \notin R_{++}^{nl}$. This implies $z_i \notin R_{++}^l$ for some i. IMR implies that there exist some $\succsim_E \in D(\succsim)$ and some $\pi \in \Pi$ such that $(z_i, i) \succsim_E (\omega_{\pi(i)}, \pi(i))$. On the other hand, the boundary condition together with the continuity and monotonicity of $\succsim_i$implies $0 \sim_i z_i$. Thus the axiom of identity implies $(z_i, i) \sim_E (0, i)$, which together with D.2 and the axiom of identity requires $(\omega_{\pi(i)}, \pi(i)) \succ_E (\frac{\omega_{\pi(i)}}{2}, \pi(i)) \succsim_E (z_i, i)$, which is a contradiction. Now we can assume $z \in R^{nl}_{++}$. Then SE implies $z \in PO(\succsim)$, and hence there exists some $p \in int.\Delta^l$ such that p is the supporting price at z. LI implies $z \in f(\succsim^p)$, which together with Lemma 2 implies $z \in IW(\succsim^p)$ and hence $z \in IW(\succsim)$, the desired result. ■

Proof of Example 5. Let us show SN. For this, it suffices to show that if $z \in \Omega(\succsim)$ and $z' \in Z$ is equally as just as z for all $\succsim_E \in D(\succsim)$, then $z' \in \Omega(\succsim)$. Take $p \in int.\Delta^l$ arbitrarily. There exist some π and ρ such that $z_i \sim_i \omega_{\pi(i)}$ and $\min\{pq : q \sim_i z_i\} = \min\{pq : q \sim_{\rho(i)} z'_{\rho(i)}\}$ for all i. Thus $\min\{pq : q \sim_{\rho(i)} z'_{\rho(i)}\} = \min\{pq : q \sim_i z_i\} = \min\{pq : q \sim_i \omega_{\pi(i)}\}$. As i and $\rho(i)$ have the same preference, this implies $z'_{\rho(i)} \sim_{\rho(i)} \omega_{\pi(i)}$, i.e., $z'_i \sim_i \omega_{\rho^{-1}(\pi(i))}$, which means $z' \in \Omega(\succsim)$.

Next, we show ELI. Let $\succsim_E \in D(\succsim)$ and $p \in int.\Delta^l$ be such that $\succsim_E$ is essentially identical to $\succsim^p_E$around $z \in Z \cap R^{nl}_{++}$. It suffices to show $z \in \Omega(\succsim) \iff z \in \Omega(\succsim^p)$. If $z \in \Omega(\succsim)$, then there exists some $\pi \in \Pi$ such that $z_i \sim_i \omega_{\pi(i)} \forall i$. As $\succsim_E$ is essentially identical to $\succsim^p_E$around z, Lemma 3 shows $pz_i \leq p\omega_{\pi(i)} \forall i$. Thus $pz_i = p\omega_{\pi(i)} \forall i$ and hence $z \in \Omega(\succsim^p)$. If $z \in \Omega(\succsim^p)$, then for some π, $pz_i = p\omega_{\pi(i)} \forall i$. As $\succsim_E$ is essentially identical to $\succsim^p_E$ around z, Lemma 3 shows that z is a Walrasian allocation for $\succsim$ with the endowments of ω^π. Thus $z \in \Omega(\succsim)$. ■

5 Conclusion

In this paper, we combined two studies in social choice, the studies of interpersonal comparisons of welfare and that of axiomatic analysis of resource allocation problems, both of which had advanced independently of each other.[14] We conclude with one more remark below.

[14] Chamber and Hayashi (2017) proposed an alternative axiomatic approach of the Walras rule that can cope with income distribution problems.

It seems to have not very much been investigated so far to what extent it is possible to create extended preferences from a given set of preference profiles and what they are.[15] In most of the literature on interpersonal comparisons of welfare, extended preferences are given a priori, and not asked for the ground.[16] In contrast, we have stated that there must be a convincing basis for the extended preferences created from a given preference profile. D.1-D.3 is the basis. We hope that this paper will provide new insights into research in this direction.

References

[1] K. J. Arrow, "Social Choice and Individual Values," New York: John Wiley, Second Edition,1963.

[2] Blackorby C, Bossert W, Donaldson D (2002) Utilitarianism and the theory of justice. In: Arrow KJ, Sen AK, Suzumura K (eds) Handbook of social choice and welfare. Elsevier, Amsterdam, pp 543–596

[3] Blackorby C, Donaldson D, Weymark JA (1984) Social choice with interpersonal utility comparisons: a diagrammatic introduction. Int Econ Rev 25:327–356

[4] Bossert W, Weymark JA (2004) Utility in social choice. In: Barber, Hammond P, Seidl C (eds) Handbook of utility theory, vol 2. Kluwer, Dordrecht, pp 1099–1177

[5] C.P. Chambers and T. Hayashi (2017), Resource allocation with partial responsibilities for initial endowments, *Int. J. Economic Theory*, Vol.13, pp355-368.

[15] Refer to Hammond (1989), a comprehensive survey of this topic, for more details.

[16] Refer to d'Aspremont (1985) for example, which defines an extended utility profile U, a numerical function defined on $X \times N$, where X is the set of alternatives, finite or infinite, unstructured. A social welfare functional, a generalization of Arrow's social welfare function, is defined on $\mathcal{U}$, the set of *all logically possible* extended utility profiles. We can see the same setting in almost every literature from Sen (1970) to Yamamura (2017).

[6] d'Aspremont C (1985) Axioms for Social welfare orderings.In: Hurwicz L, Schmeidler D, Sonnenschein H (eds) Social goals and social organization: essays in memory of Elisha Pazner. Cambridge University Press, Cambridge, pp 19–67

[7] d'Aspremont C and Gevers L (1977) Equity and informational basis of collective choice, Review of Economic Studies Vol.44, pp199-209

[8] d'Aspremont C and L. Gevers (2002) Social welfare functionals and interpersonal comparability in Handbook of Social Choice and Welfare Vol.1 Chapter 10 459-541.

[9] Deschamps R, Gevers L (1978) Leximin and utilitarian rules: a joint characterization. J. Econ. Theory 17:143–163

[10] Fleurbaey M, Hammond PJ (2004) Interpersonally comparable utility. In: Barber S, Hammond P, Seidl C (eds) Handbook of utility theory, vol 2. Kluwer, Dordrecht, pp 1179–1285

[11] M. Fleurbaey, K. Suzumura, and K. Tadenuma, (2005) The informational basis of the theory of fair allocation. *Social Choice and Welfare* **24**, pp.311–341.

[12] Gevers L (1979) On interpersonal comparability and social welfare orderings. Econometrica 44:75–90

[13] Gevers L (1986) Walrasian social choice: some simple axiomatic approaches. In: Heller et al.(eds) Social choice and public decision making: essays in honor of Arrow JK Vol 1. Cambridge Univ. Press, Cambridge

[14] Hammond JP (1976) Equity, Arrow's conditions, and Rawls' difference principle, Econometrica Vol.44 pp793-804

[15] J.P. Hammond (2010), Competitive market mechanisms as social choice procedures, *in* "Handbook of Social Choice and Welfare Vol.2," K. J. Arrow, A. Sen, and Kotaro Suzumura Eds.), pp 47-152, Amsterdam: Elsevier.

[16] Hammond JP (1991) Interpersonal comparisons of utility: Why and how they are and should be made, in "Interpersonal Comparisons of Well-Being" (Elster J and JE Roemer Eds.) pp200-254. Cambridge Univ. Press

[17] Hare, R. M. (1981). Moral thinking: its levels, method, and point. Oxford University Press.

[18] Harsanyi, J. C. (1955) "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility," Journal of Political Economy 63: 309-321

[19] L. Hurwicz, On allocations attainable through Nash equilibria. *J. Econ. Theory* **21**, (1979), pp.140-165.

[20] Maniquet F (1996) Horizontal equity and stability when the number of agents is variable in the fair division problem Economics Letters Vol. 50: 85-90

[21] K. Miyagishima, (2015) Implementability and equity in production economies with unequal skills, *Rev. Econ. Design* **19**, pp.247–257.

[22] R. Nagahisa, (1991) A local independence condition for characterization of Walrasian allocations rule, *J. Econ. Theory* **54**, pp.106-123.

[23] R. Nagahisa and SC. Suh, (1995) A characterization of the Walras rule, *Soc. Choice Welfare* **12**, pp.335-352; reprinted *in* "The Legacy of Léon Walras Vol.2: Intellectual Legacies in Modern Economics 7" (D. A. Walker, Ed.), pp. 571-588, Cheltenham (UK): Edward Elgar Publishing Ltd, 2001.

[24] Rawls J (1971) A Theory of Justice Harvard University Press

[25] Roberts K (2010) Social choice theory and the informational basis approach. In: Morris CW, Sen A (eds) Cambridge University Press, Cambridge, pp 115–137

[26] Sakai, T (2009) Walrasian social orderings in exchange economies, *J. Math. Econ.* **45**, pp.16–22.

[27] Sen AK (1970) Collective Choice and Social Welfare Holden-Day: San Francisco.

[28] Sen AK (1974a) Rawls versus Bentham: An axiomatic examination of the pure distribution problem, Theory and Decision 4 301-309.

[29] Sen AK (1974b) Informational bases of alternative welfare approaches: Aggregation and income distribution, Journal of Public Economics 3 387-403.

[30] Sen AK (1976) Welfare inequalities and Rawlsian axiomatics. Theory Dec 7: 243–262

[31] Sen AK (1977) On weights and measures: Informational constraints in social welfare analysis, Econometrica 45 pp1539-72

[32] Sen AK (1986) Social choice theory. In: Arrow KJ, Intriligator MD (eds) Handbook of mathematical economics, vol 3. North-Holland, Amsterdam, pp 1073–1181

[33] Strasnick S (1976) Social choice and the deviation of Rawls' difference principle, Journal of Philosophy 73 pp184-194

[34] Suppes P (1966) Some formal models of grading principles. Synthese 6: 284-306

[35] Suzumura K (1983) Rational Choice, Collective Decisions, and Social Welfare, Cambridge Univ. Press.

[36] W. Thomson, (1988) A study of choice correspondences in economies with a variable number of agents, *J. Econ. Theory* **46**, pp.237-254.

[37] M. Toda, (2004) Characterizations of the Walrasian solution from equal division, mimeo.

[38] K. Urai and H. Murakami, (2015) Local Independence, monotonicity and axiomatic characterization of price-money message mechanism, mimeo.

[39] H. Yamamura, (2017) Interpersonal comparison necessary for Arrovian aggregation, Social Choice and Welfare 49:37–64

[40] N. Yoshihara, (1998) Characterizations of public and private ownership solutions, *Math. Soc. Science* **35**, pp.165–184.