Profit Share Equilibria for Economies with Public Goods

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2019 年 8 月 7 日
数理経済学会 方法論分科会
近畿地区 夏季ジョイントセミナー

Contents

Introduction (1)

- 公共財の最適供給問題と Lindahl 均衡

公共財の存在する経済において、パレート最適な資源配分を市場メカニズムを通じて実現する方法については、Lindahl (1919) あるいは Samuelson (1954) に至るまで古くから議論されてきた。一般に公共財は、非競合性ならびに非排除性を有するため、正の外部性がある場合にはその供給量が過少になり、負の外部性がある場合にはその供給量が過大になることが知られている（市場の失敗）。

Lindahl 均衡とは、もし各主体の公共財への選好が誤りなく表明されるならば、各人に向けた個別の価格を用いた仮想的な市場を通じて、その最適な資源配分を実現できるという考え方である。

Introduction (2)

- Lindahl 均衡と一般均衡モデル

Lindahl 均衡の考え方に基づき、Arrow-Debre 的な一般均衡理論の枠組みにおいて定式化し直されたものが、Lindahl-Foley 均衡である。この均衡は、公共財生産の利潤や私企業の取扱い、財の種類や satiation の有無の問題、国家による財政支出の捉え方等の点において、改良が求められる概念である。

• Foley (1970) · · · 私的財・公共財ともに複数であるが、コアの問題に関しては生産技術を cone に限定（⇒ 企業の利潤は常にゼロになる）。Free disposability を仮定すれば別（Milleron 1972）。

• Mas-Colell and Silvestre (1989) · · · コア性の問題等において、企業の利潤が正となり得る場合を扱うが、私的財が 1 財（ニュメレール）のみであることに結果が強く依存。

Introduction (3)

- Dividend Lindahl Equilibrium と本稿の目的

本稿の Dividend Lindahl Equilibrium は Mas-Colell and Silvestre (1989) の Lindahl-Foley 均衡を踏襲し、費用負担 cost-share よりは、利潤の分配 profit-share に重点を置く均衡概念である。

$*$ 私的財は複数、satiation あり

$*$ 私企業は複数で、公共財の費用を私企業も負担する

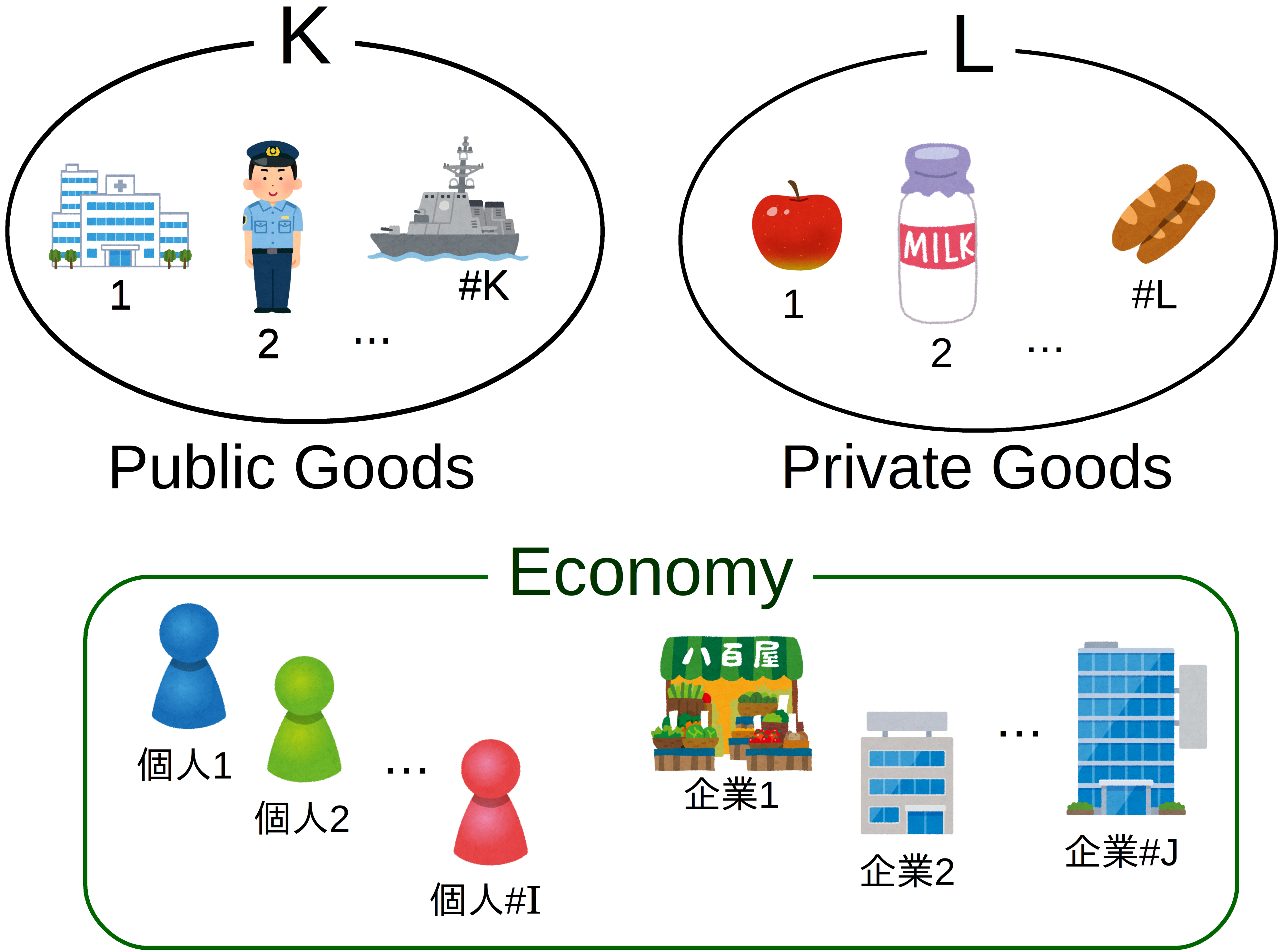
$*$ 公企業の利潤も考慮する

この均衡概念に対し、コア収束定理等を示すことで、公共財を含む経済における資源配分の最適性や安定性、あるべき資源配分を実現する市場メカニズムの可能性についての検討を行う。

Mathematical Notations

- R is the set of real numbers.
- $K \cup L$ is the finite set of commodities.
  $\cdots$ $K \neq \emptyset$ is the index set of **public goods**.
  $\cdots$ $L \neq \emptyset$ is the set of **private goods** $(K \cap L = \emptyset)$.
- I is the index set of **agents** (=consumers).
- J is the index set of **private firms**.

For finite set A, denote by $\sharp A$ the number of elements of A. Write R^K instead of $R^{\sharp K}$ to represent $\sharp K$-dimensional vector space.

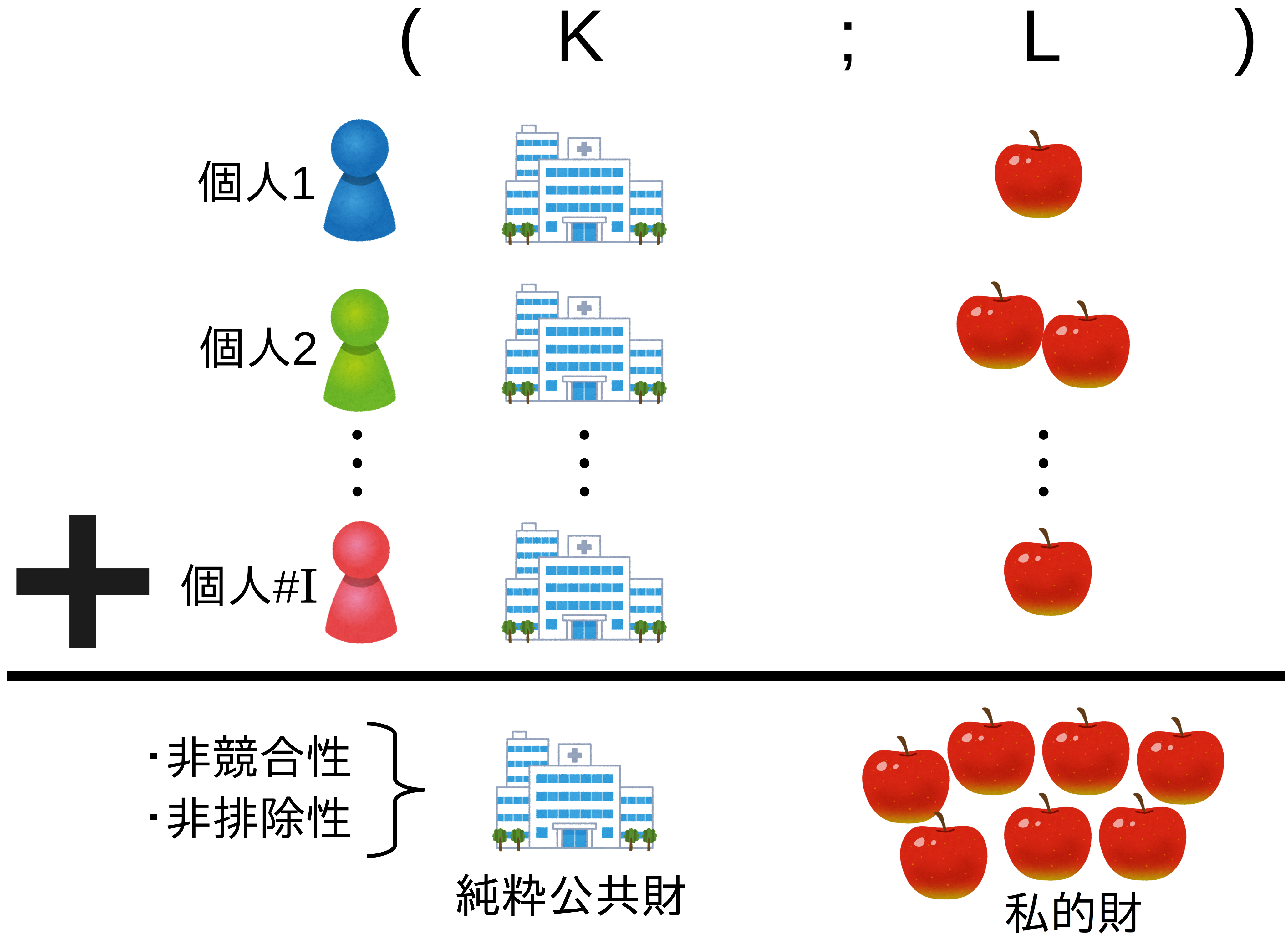

K
1
2
…
#K
Public Goods
L
MILK
1
2
…
#L
Private Goods
Economy
個人1
個人2
…
個人#I
八百屋
企業1
企業2
…
企業#J

Model (1)

As in Foley (1970), a vector of public and private goods is written as $(x_1, \ldots, x_{\sharp K}; z_1, \ldots, z_{\sharp L}) = (x; z) \in R^{K \cup L}$.

Each agent $i \in I$ is written by $(\succsim_i, \omega_i)$, where $\succsim_i$ is preference relation on closed convex consumption set $X_i \subset R^{K \cup L}$ bounded from below, and $\omega_i \in R^L_{++}$ is the initial endowment. The preferences are assumed to satisfy reflexivity, transitivity, completeness, continuity, and strict convexity.

For each agent, preference can be **satiated**, and locally non-satiated at every point except for the maximal satiation point.


(K ; L)
個人1
個人2
個人#I
・非競合性
・非排除性
純粋公共財
私的財

Model (2)

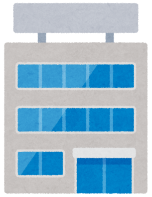
An economy $\mathcal{E}$ is a finite lists of agents, $(\succsim_i, \omega_i)_{i \in I}$, private firm technologies, $(Y_j)_{j \in J}$, and a public firm technology Y_0.

(1-1) For each $j \in J$, Y_j is a closed convex subset of $R^{K \cup L}$ s.t. Y_j has no positive coordinate for public goods.

(1-2) Technology Y_0 is a closed convex subset of $R^{K \cup L}$ s.t. $0 \in Y_0$ and Y_0 has no positive coordinate for private goods.

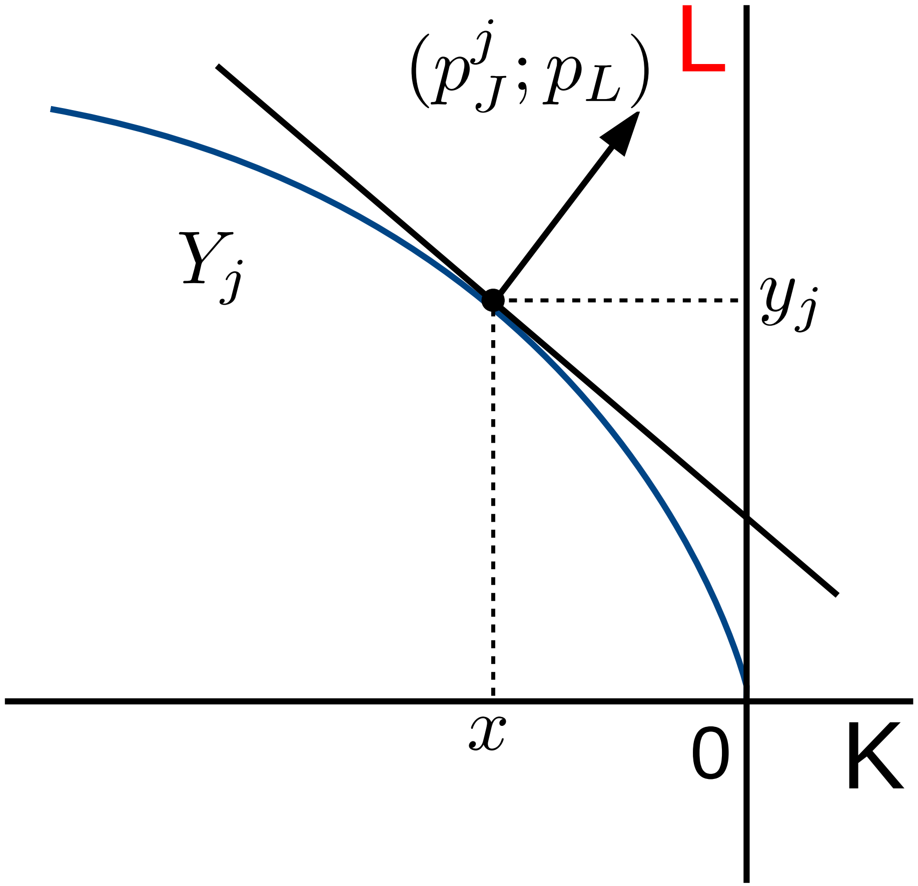
(1-3) For each Y_j, public goods are harmless in the sense that if $(x; z) \in Y_j$, then $(x'; z) \in Y_j$ for all $x' \geqq x$.

N.B. Compare (1-3) with the condition in Foley (1970), i.e., (B.5) No public good is necessary as a production input.



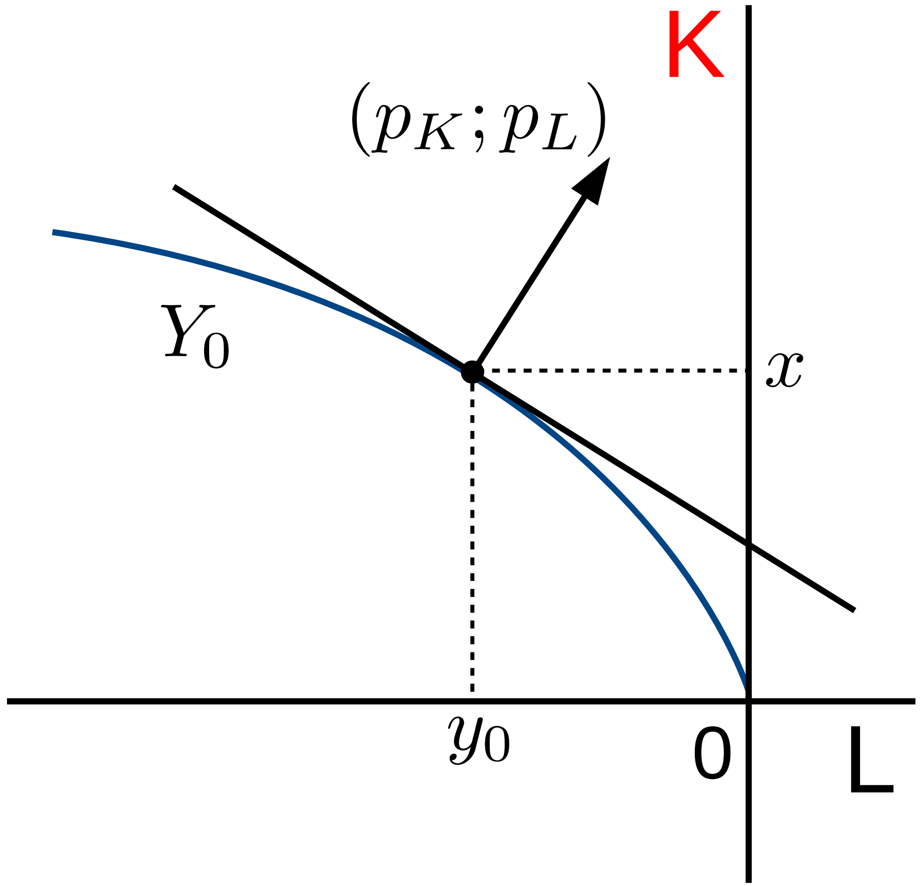
私企業 j

(j=1, 2,･･･, #J)



公企業 0

（公共財の生産技術）



Model (3)

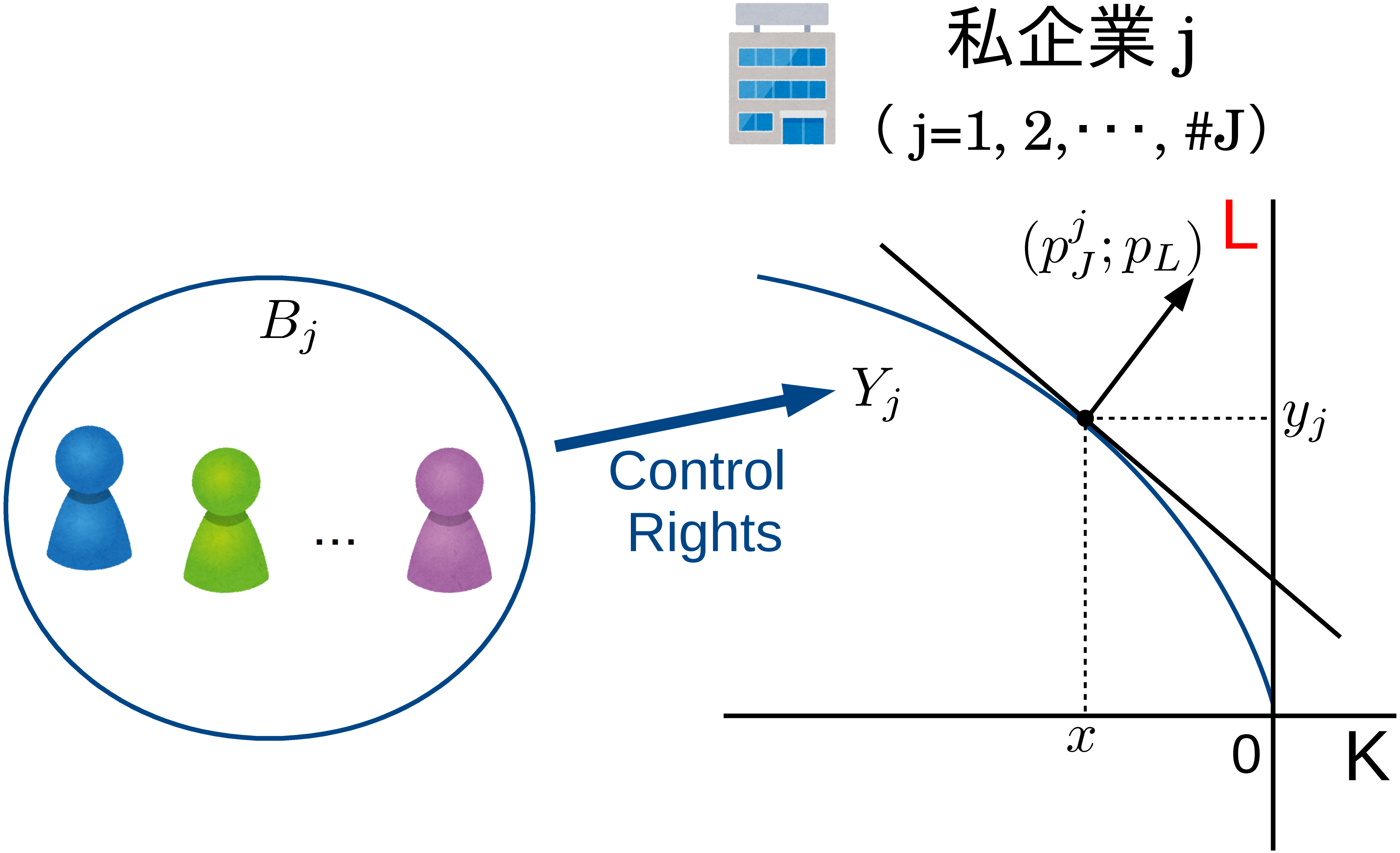
The set of all technically possible production plans in the economy, Y, is defined as the following set:

$Y = \{(x; y) | y = y_0 + \sum_{j \in J} y_j, (x; y_0) \in Y_0, (x; y_j) \in Y_j$ for all $j \in J\}$.

For private firms $(Y_j)_{j \in J}$, the list of sets of the **technology owners** is represented by $(B_j)_{j \in J}$.

Y and Y_0 are owned by I, the set of all members of economy.

The pure exchange case is $Y_j = \{0\}$ and $B_j = \emptyset$ for all $j \in J$.



私企業 j
(j=1, 2,・・・, #J)
B_j
...
Control Rights
Y_j
$(p^j_J; p_L)$
L
y_j
x
0
K

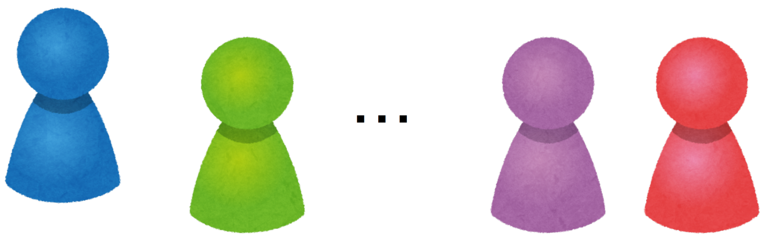
価値財（R. Musgrave）は、人間社会のあるべき姿 – それが国家によって異なるなら個人の信条への侵害? – という目的に向けられた財として、それを普遍的かつ十全に行き渡らせる役割を国家が担うという意味で、目指された公共財であるといえる。

公企業 0

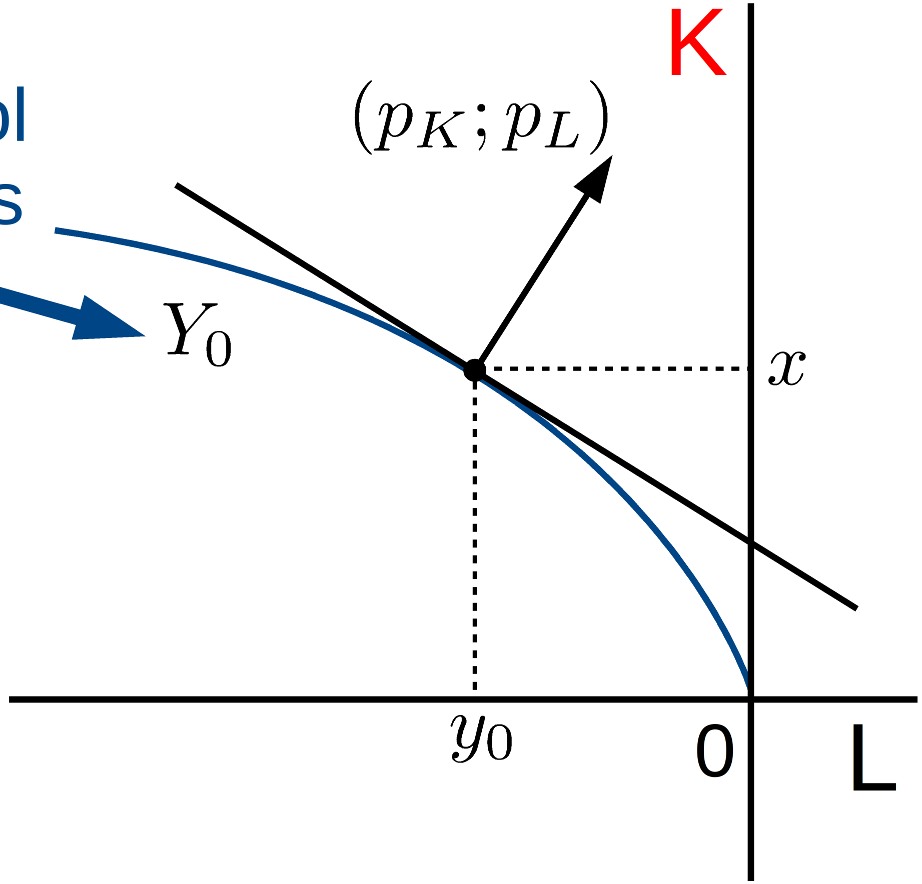
（公共財の生産技術）

公共財 x は全員に（非排除性）、等量消費される（非競合性）。

Closed Production Group 等のオートノミー　C



Control Rights



※オートノミー：ここでは、status quo allocation を、自分たちだけの生産技術でまかなえる集団

Model (4)

A **consumption allocaion** for a list of agents I is a sequence of elements of consumption sets, $((x_i; z_i) \in X_i \subset R^{K \cup L})_{i \in I}$.

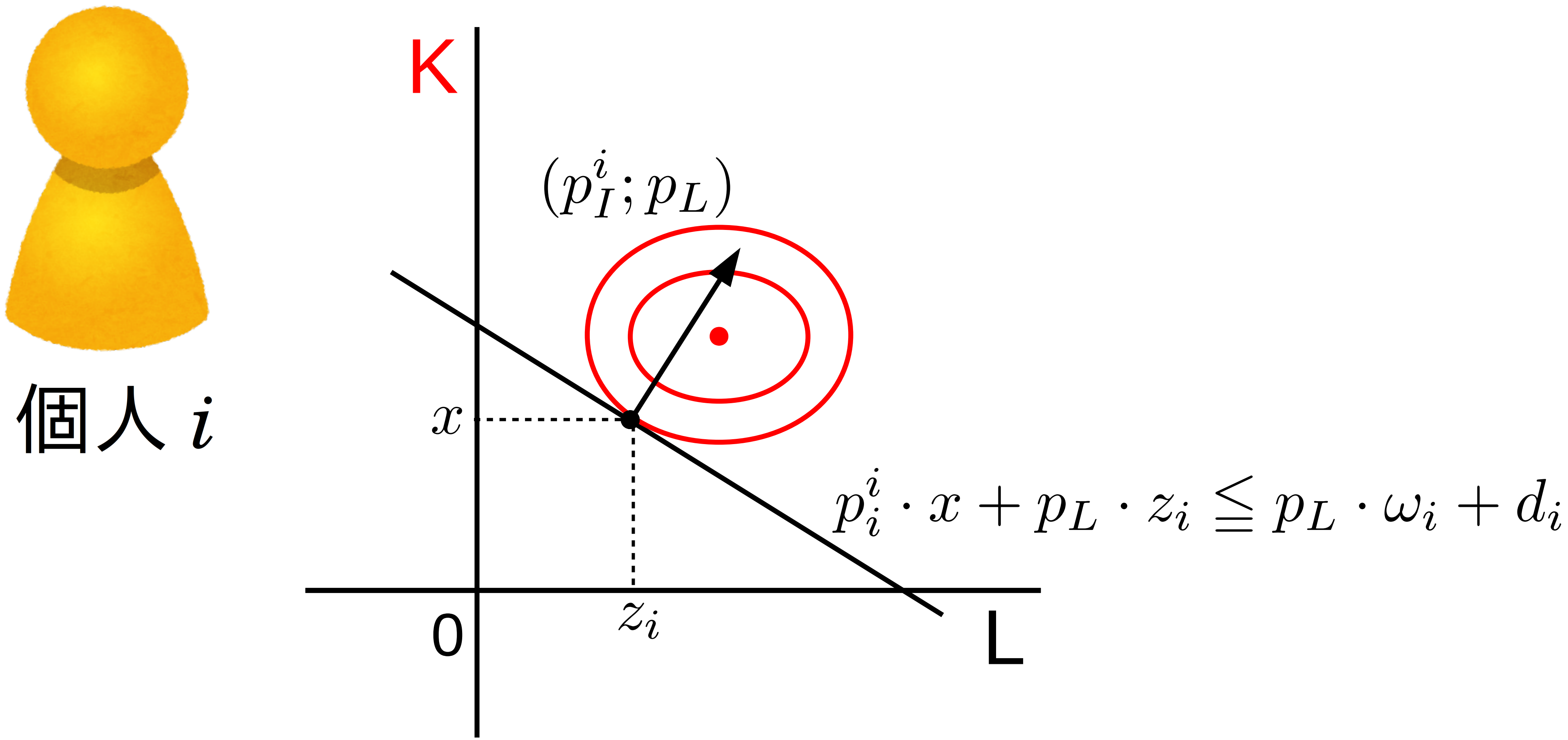
For economy $\mathcal{E} = ((\succsim_i, \omega_i)_{i \in I}, Y, (B_j)_{j \in J})$, if $(x; z_i)_{i \in I}$ satisfies

$$\sum_{i \in I} z_i = \sum_{i \in I} \omega_i + y \text{ for some } (x; y) \in Y,$$

we say that $(x; z_i)_{i \in I}$ is **feasible** under $(x; y) \in Y$.

(K ; L)

$(\quad x \quad ; \quad z_i \quad)$



Dividend Lindahl Equilibrium

Dividend Lindahl equilibrium for economy $\mathcal{E}$ is the list of

- **price vector** $p^* = (p_K^*; p_L^*) \in R^{K \cup L}$, private price vectors for consumers $(p_I^{i\,*} \in R^K)_{i \in I}$ and producers $(p_J^{j\,*} \in R^K)_{j \in J}$ such that $p_K^* = \sum_{i \in I} p_I^{i\,*} + \sum_{j \in J} p_J^{j\,*}$,
- non-negative **dividends** $d^* = (d_i^*)_{i \in I} \in R_+^I$,
- **feasible consumption allocation** $(x^*; z_i^*)_{i \in I}$ under $(x^*; y^*) \in Y = Y_0 +^{(L)} \sum_{j \in J}^{(L)} Y_j$ with $y^* = y_0^* + \sum_{j \in J} y_j^*$, where $(x^*; y_0^*) \in Y_0$ and $(x^*; y_j^*) \in Y_j$ for each $j \in J$,

which satisfies the following three conditions:

(i) $p_L^* \cdot y_j^* - p_J^{j\,*} \cdot x^* \geqq p_L^* \cdot y_j - p_J^{j\,*} \cdot x$ for all $(x; y_j) \in Y_j$ and $j \in J$,

(ii) $p^* \cdot (x^*; y_0^*) \geqq p^* \cdot (x; y_0)$ for all $(x; y_0) \in Y_0$,

(iii) for each $i \in I$, $(x^*; z_i^*)$ is the $\succsim_i$-greatest element in the set,

$$\{(x; z_i) \in R^{K \cup L} \mid p_I^{i\,*} \cdot x + p_L^* \cdot z_i \leqq p_L^* \cdot \omega_i + d_i^*\}.$$

Lemma 1

Lemma 1 A dividend Lindahl equilibrium satisfies

$$\sum_{i\in I} d_i^* = \sum_{j\in J}(p_L^* \cdot y_j^* - p_J^{j\,*} \cdot x^*) + p^* \cdot (x^*; y_0^*),$$

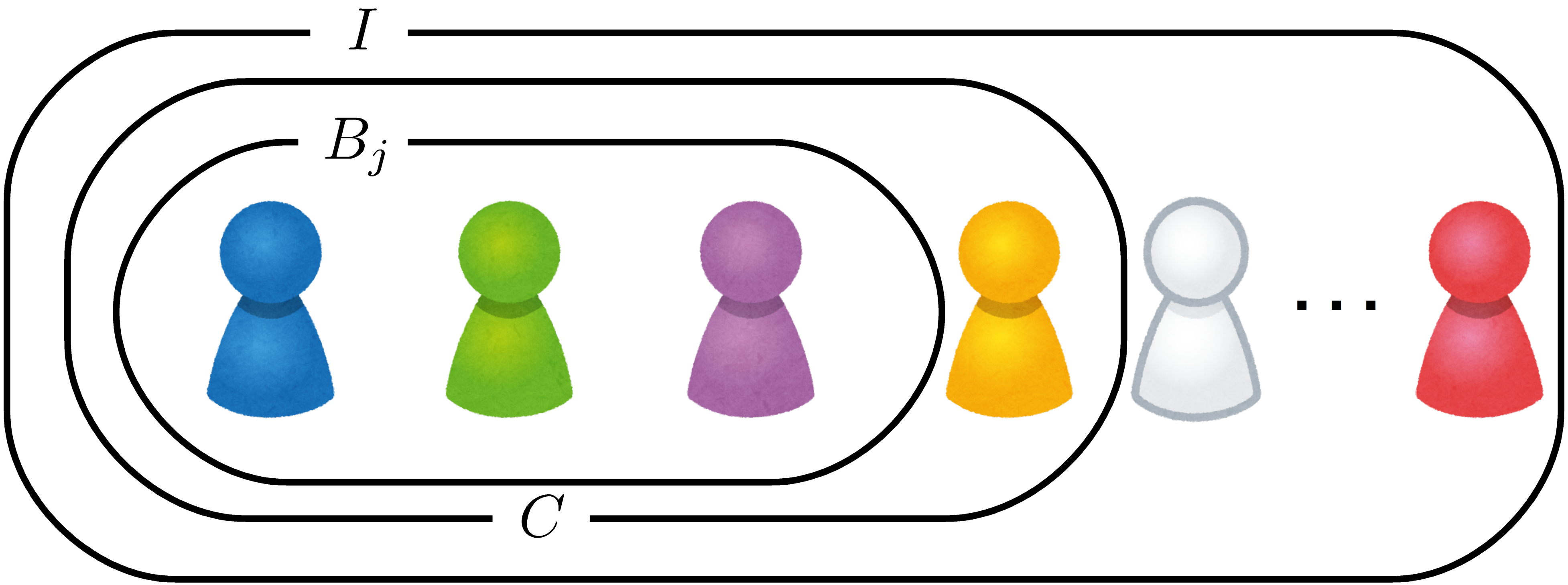
if the equilibrium allocation has **no satiation point**. In general, by the feasibility condition and the budget constraint (iii), we have $\sum_{i\in I} d_i^* \geqq \sum_{j\in J}(p_L^* \cdot y_j^* - p_J^{j\,*} \cdot x^*) + p^* \cdot (x^*; y_0^*)$,

Profit は全員で分配しているが、公企業が赤字であれば、私企業の利潤が分配し尽くされる必要はない（むしろし尽くされると、Walras Law に矛盾する）。

Dividend Lindahl Equilibrium (Profit Share)

$$\sum_{i\in C} d_i \geqq \sum_{j\in J(C)} (p_L^* \cdot y_j^* - p_J^{j\,*} \cdot x^*) + (\sum_{i\in C} p_I^{i\,*} + \sum_{j\in J(C)} p_J^{j\,*}; p_L^*) \cdot (x^*; y_0^*)$$

・・・Profit は Closed Production Group C で分けている。



Dividend Lindahl Equilibrium (2)

Dividend Lindahl Equilibrium is a generalized concept of **Lindahl-Foley Equilibrium** of Mas-Colell and Silvestre (1989) in the following sense:

- There are **multiple** private goods and public goods.
- Preferences are possibly **satiated**.
- There are **multiple** private firms. Consumers and **private frims** pay the **cost of public goods**.
- Public firm **maximizes** its **profits**. Our equilibrium concept focuses on the **profit-shares** of public and private firms.

Closed Production Group (1)

Given feasible $(x^*; z_i^*)_{i \in I}$ under $(x^*; y_0^*) \in Y_0$ and $(x^*; y_j^*) \in Y_j$ for each $j \in J$ in $\mathcal{E}$, a group of members of I, $C \subset I$, is called a **closed production group** for $(x^*; z_i^*)_{i \in I}$ under $(x^*; y^*)$ with $y^* = y_0^* + \sum_{j \in J} y_j^*$ if there exists $J(C) = \{j \in J \mid B_j \subset C\}$ s.t.

$$\sum_{i \in C} z_i^* = \sum_{i \in C} \omega_i + y_0^* + \sum_{j \in J(C)} y_j^*,$$

where $(x^*; y_0^*) \in Y_0$ and $(x^*; y_j^*) \in Y_j$ for all $j \in J(C)$.

For closed production group C, their **status quo** allocations of **public** and **private** goods can be supplied through public goods technology Y_0, private firms' technologies $\sum_{j \in J(C)} Y_j$ owned merely by members of C, and resources owned by C.

Closed Production Group (2)

The total members group I of $\mathcal{E}$ is always a closed production group. It is quite natural to consider that such a group has **full control rights** to change their production $\sum_{j \in J(C)} y_j^*$ to another.

From the definition of CPG and the budget constraint (iii) of dividend Lindahl equilibrium, we have the next lemma.

Lemma 2 For each closed production group $C \subset I$, $\sum_{i \in C} d_i^* \geqq \sum_{j \in J(C)} (p_L^* \cdot y_j^* - p_J^{j\,*} \cdot x^*) + (\sum_{i \in C} p_I^{i\,*} + \sum_{j \in J(C)} p_J^{j\,*}; p_L^*) \cdot (x^*; y_0^*)$.

$\cdots$ All profits are completely distributed in each **CPG**.

Admissible Production Decision Groups

Suppose that $\mathcal{E} = ((\succsim_i, \omega_i)_{i \in I}, Y, (B_j)_{j \in J})$ is associated with the structure of production decision groups for **private goods**.

Admissible production decision groups $\mathfrak{C}(\mathcal{E}, (x; z_i)_{i \in I}, y) = (C_h)_{h \in H}$ for each $(x; z_i)_{i \in I}$ feasible under $(x; y)$ with $y = y_0 + \sum_{j \in J} y_j$, $(x; y_0) \in Y_0$, $(x; y_j) \in Y_j$ for each $j \in J$, of $\mathcal{E}$ as follows:

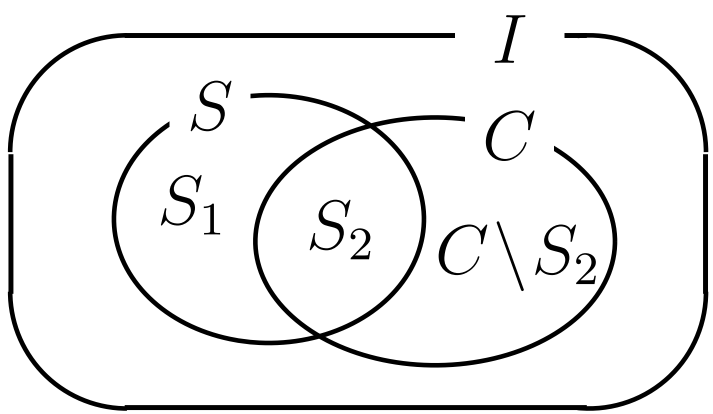
- $\emptyset$ and I are elements of $\{C_h \mid h \in H\}$,
- for every $h \in H$, there is a private technology, $Y_{C_h} \subset R^{K \cup L}$,
- if C_h is CPG, $Y_{C_h} = \{(x; y) | y = \sum_{j \in J(C_h)} y_j, (x; y_j) \in Y_j\}$.

Public Production Core

Given economy $\mathcal{E} = ((\succsim_i, \omega_i)_{i \in I}, Y, (B_j)_{j \in J})$, $(x; z_i)_{i \in I}$ feasible under $(x; y) \in Y$ with $y = y_0 + \sum_{j \in J} y_j$, is a **public production core** allocation with $\mathfrak{C}(\mathcal{E}, (x; z_i)_{i \in I}, y) = (C_h)_{h \in H}$, if there are no coalition S with partition (S_1, S_2), $S = S_1 \cup S_2$, where $S_1 \subset I \setminus C$ and $S_2 \subset C$ for a certain $C = C_h$, $h \in H$, and no allocation $(x'; z'_i)_{i \in S \cup C}$ for $S \cup C$ with $(x'; y') \in Y_0 +^{(L)} Y_{C_h}$, where $y' = y'_0 + y'_{C_h}$, satisfying the following conditions:

(f-1) $\sum_{i \in S \cup C} z'_i = y'_0 + y'_{C_h} + \sum_{i \in (S \cup C)} \omega_i$,

(g) $(x'; z'_i) \succ_i (\bar{x}; \bar{z}_i)$ for all $i \in S$, and
$(x', z'_i) \succsim_i (\bar{x}; \bar{z}_i)$ for all $i \in C \setminus S$.



Optimality and Core [Equilibrium $\Rightarrow$ Core]

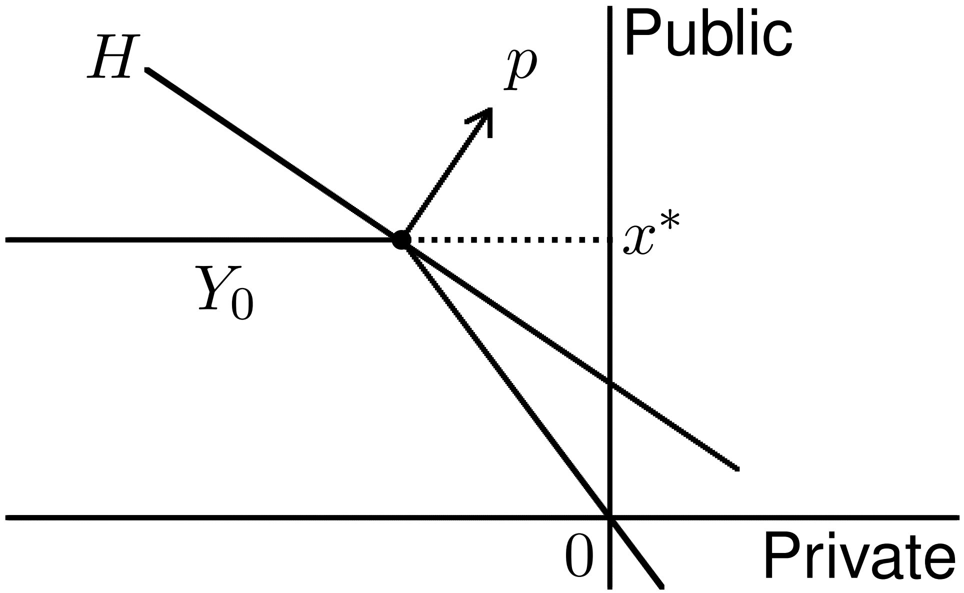
Theorem 2* If feasible allocation $(x^*; z_i^*)_{i\in I}$ under $(x^*; y^*)\in Y$ is **dividend Lindahl equilibrium** for $\mathcal{E} = ((\succsim_i, \omega_i)_{i\in I}, Y, (B_j)_{j\in J})$ under $p^* \neq 0$ with $\mathfrak{C}(\mathcal{E}, (x^*; z_i^*)_{i\in I}, y^*) = (C_h^*)_{h\in H^*}$, where C_h is CPG for $(x^*; z_i^*)_{i\in I}$ under $(x^*; y^*)$ for each $h \in H^*$, then $(x^*; z^*)^n$ is a **public production core** allocation of $\mathcal{E}^n$ for every $n \geqq 1$.

(1) **Pareto optimality** can be proved as the case with $C = I$.

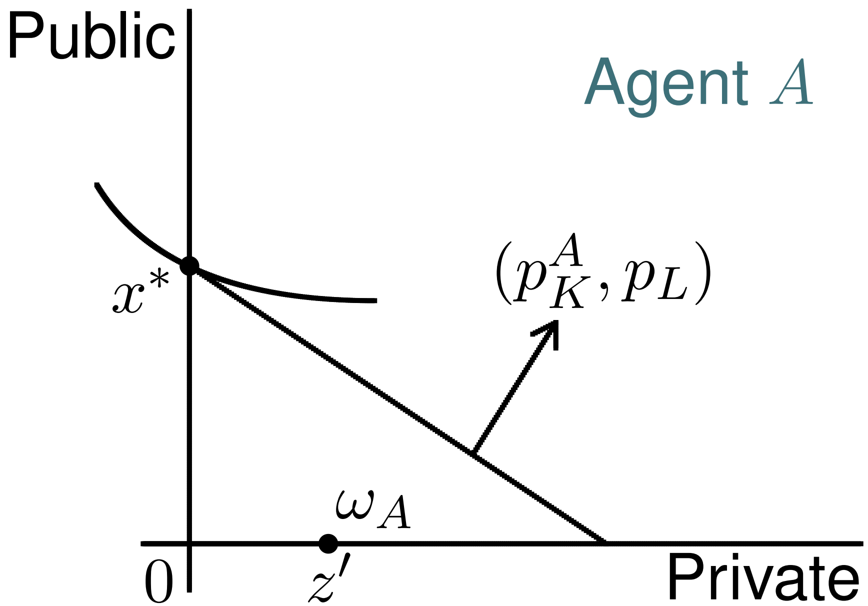
(2) If the amount of public goods is fixed, for each $n \geqq 1$, **core** property is proved when $p_L^* \cdot z_i^* \geqq p_L^* \cdot \omega_i$ for all $i \in S_1$.

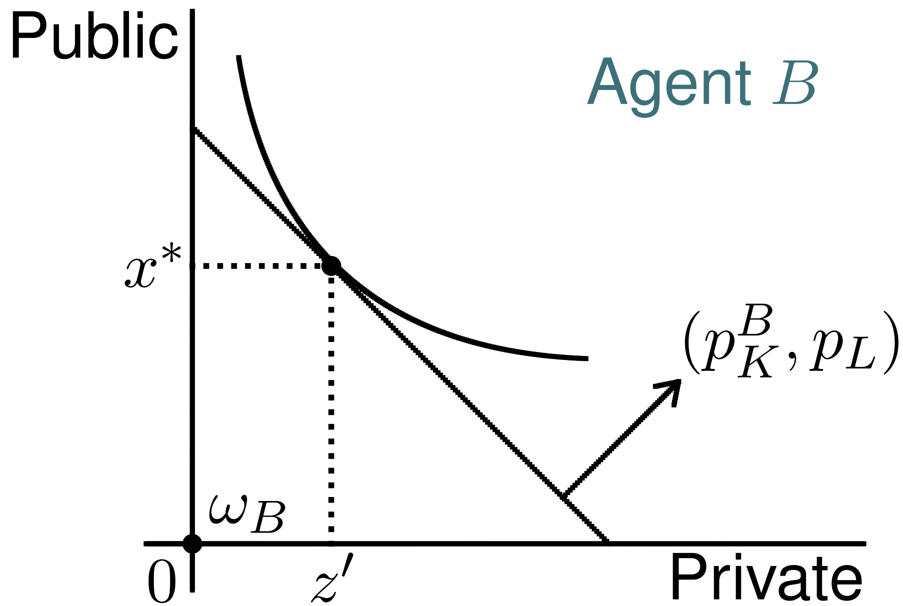
(3) If the amount of public goods is not fixed, even for $n = 1$, **core** property is difficult to prove. $\cdots$ The result of Mas-Colell and Silvestre (1989) is hard to extend.

Counter Example for Case (2) of Theorem 2



- Agent A の均衡資源配分 $\cdots$ $p_L^* \cdot z_A^* < p_L^* \cdot \omega_A$.
- Dividend Lindahl 均衡は $S_1 = \{A\}$, $C = \{A, B$ を除く I のメンバー全員 $\}$ でブロックされる。





Core Limit Theorem [Core $\Rightarrow$ Equilibrium]

Theorem 1 A feasible allocation $(x^*; z_i^*)_{i \in I}$ under $(x^*; y^*) \in Y$ with $y^* = y_0^* + \sum_{j \in J} y_j^*$ for $\mathcal{E} = ((\succsim_i, \omega_i)_{i \in I}, Y, (B_j)_{j \in J})$, with $\mathfrak{C}(\mathcal{E}, (x^*; z_i^*)_{i \in I}, y^*)_{j \in J}) = (C_h^*)_{h \in H}$, where $\{C_h^* \mid h \in H\} \supset \{I\}$, is a **dividend Lindahl equilibrium** if its n-fold replica allocation belongs to the **public production core** of $\mathcal{E}^n$ for every $n \geqq 1$.

$\cdots$ Public Production Core の意味で望ましい資源配分を絞り込んでいくと、Dividend Lindahl Equilibrium（価格と利潤および補助金の分配の下での競争均衡資源配分）が残ってくる。

Conclusion

- Dividend Lindahl Equilibrium は、「公共財の購入費は別として、政府からの移転は非負」という無理のない再配分を伴った価格メカニズムの下で実現する資源配分である。
- Dividend Lindahl Equilibrium は、パレート最適であり、いくつかの条件を追加すると、Public Production Core の Limit との同値まで言える。
- Dividend Lindahl Equilibrium は赤字財政下で実現される可能性がある（私企業の利潤と公企業の利潤の合計が正の場合でも、この赤字は生じ得ることに注意）。

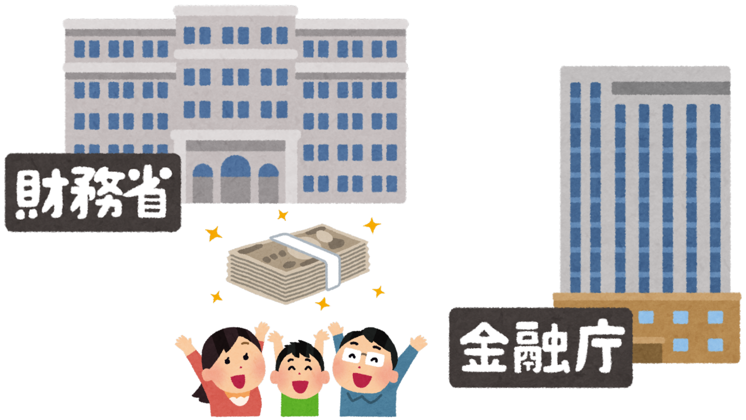
Dividend Lindahl Equilibrium の特徴

Lemma 2 For each closed production group $C \subset I$, $\sum_{i \in C} d_i^* \geqq \sum_{j \in J(C)} (p_L^* \cdot y_j^* - p_J^{j*} \cdot x^*) + (\sum_{i \in C} p_I^{i*} + \sum_{j \in J(C)} p_J^{j*}; p_L^*) \cdot (x^*; y_0^*)$.

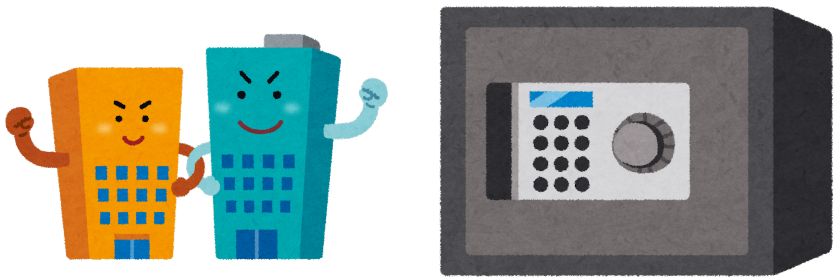
$\cdots$ Satiation がない場合、上式は等号で成立する。Profit は Closed Production Group C で分配されているが、公企業が赤字であれば、私企業の利潤が分配し尽くされる必要はない（むしろし尽くされると、Walras Law に矛盾）。

例：医療費による財政赤字と私企業

国が治療費を定める ⇒高額医療を実施 ⇒ 税で補填される



公共財としての医療を提供している国家という観点から、公企業は赤字



財政赤字（相当額）が、企業の内部留保として定着する形で均衡となっている

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