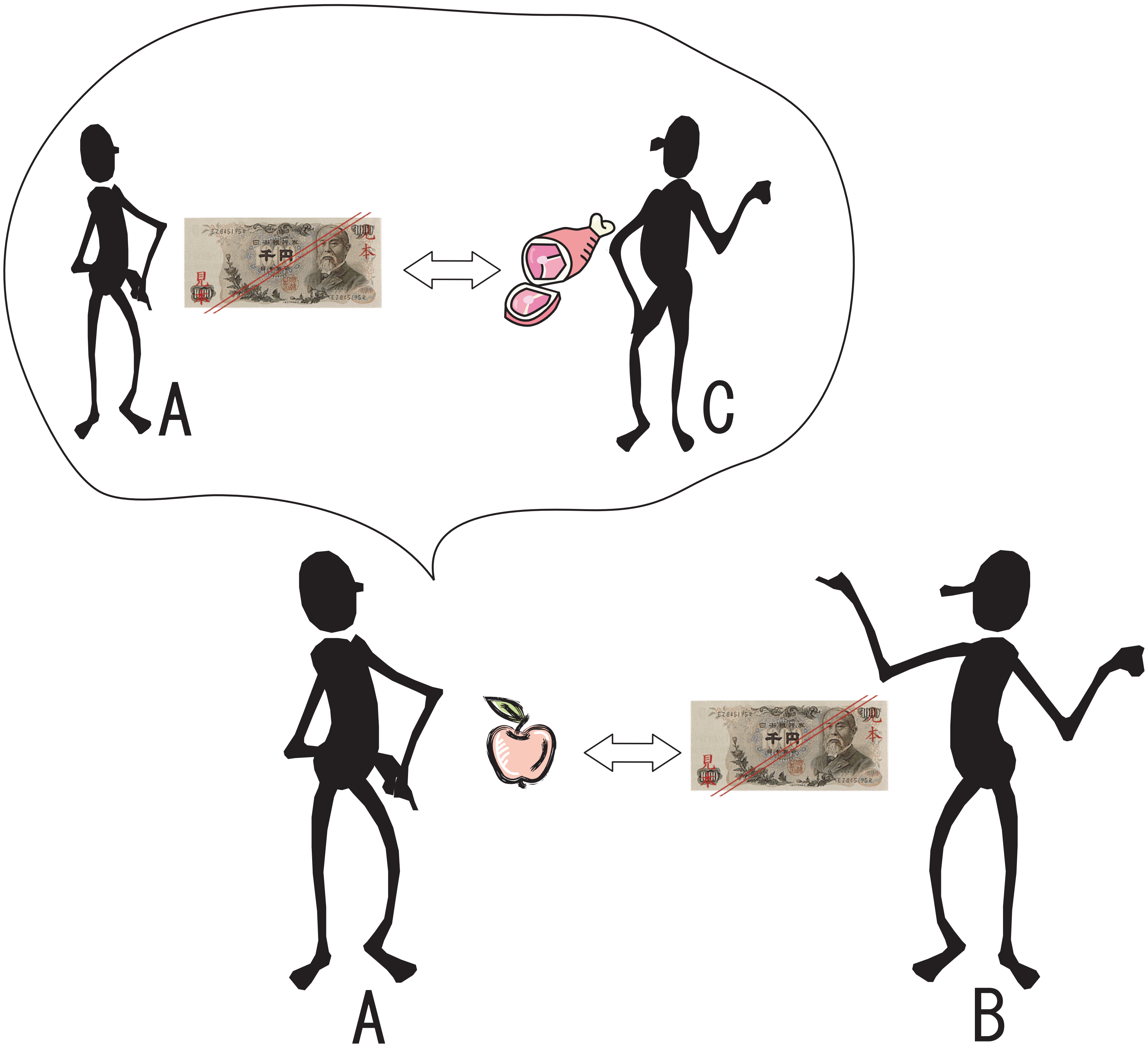
On the Indeterminacy of Monetary Equilibria

Kazuya Kamiya

Kobe University

Money has value as a medium of exchange. (Iwai, Kiyotaki and Wright)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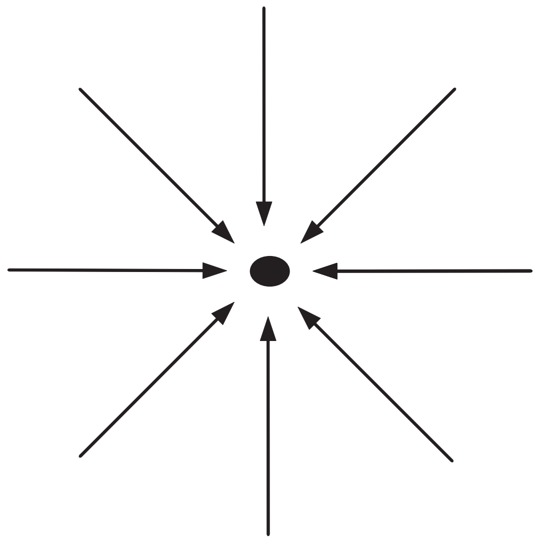
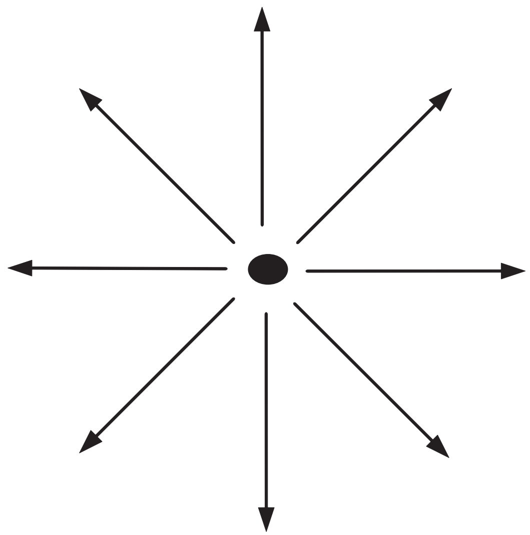
In their models, money is indivisble. More precisely, each agent can have just one unit of money.





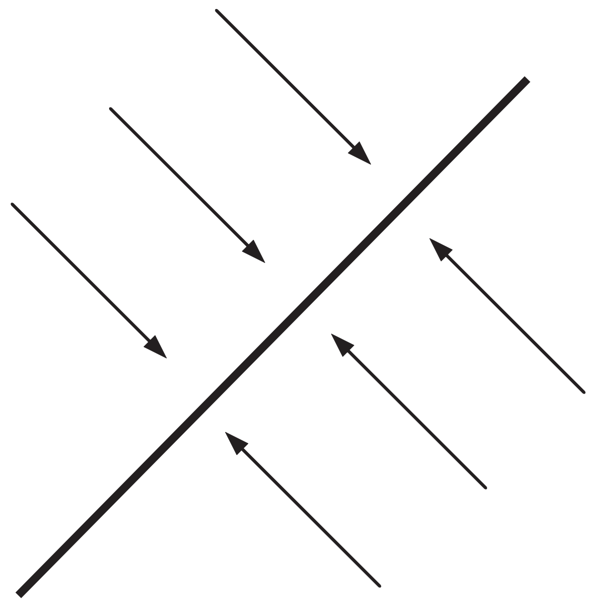
Indeterminacy of Monetary Equilibria

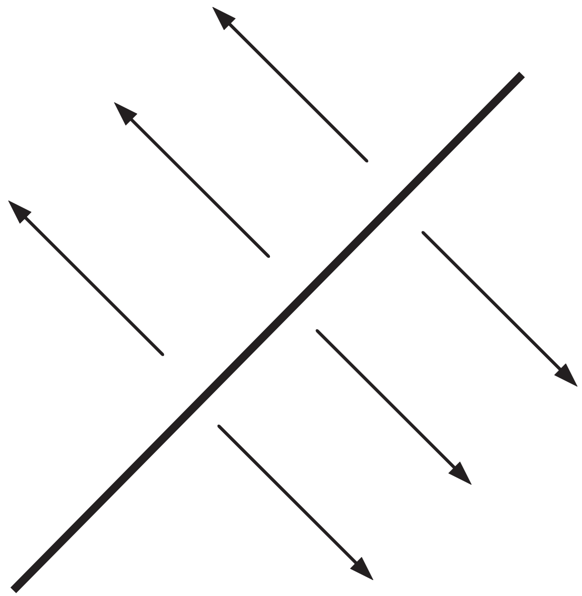
There are a finite number of stationary equilibria.

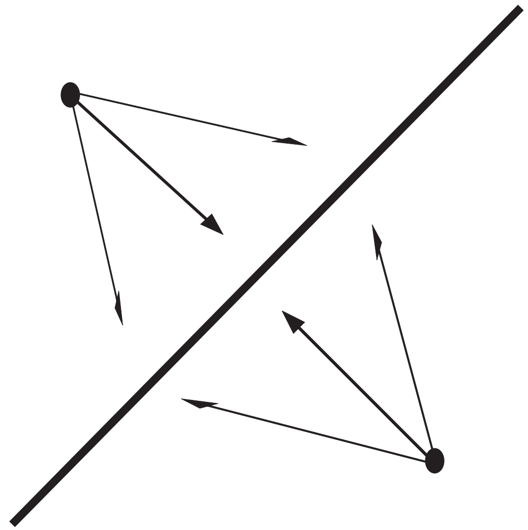
Indeterminacy of Monetary Equilibria

- In a special random matching model with divisible money, Green and Zhou (1998, 2002) show that equilibria are indeterminate, i.e., the set of equilibria is a continuum. (Real indeterminacy!)

> *"It is an open question whether this indeterminacy reflects a fundamental fact about random matching models." (Green and Zhou (2002).)*







Indeterminacy of Monetary Equilibria

- Kamiya and Shimizu (2006) show that it is an intrinsic property of random matching models. That is in any random matching model with divisible money, stationary equilibria are (generically) indeterminate.

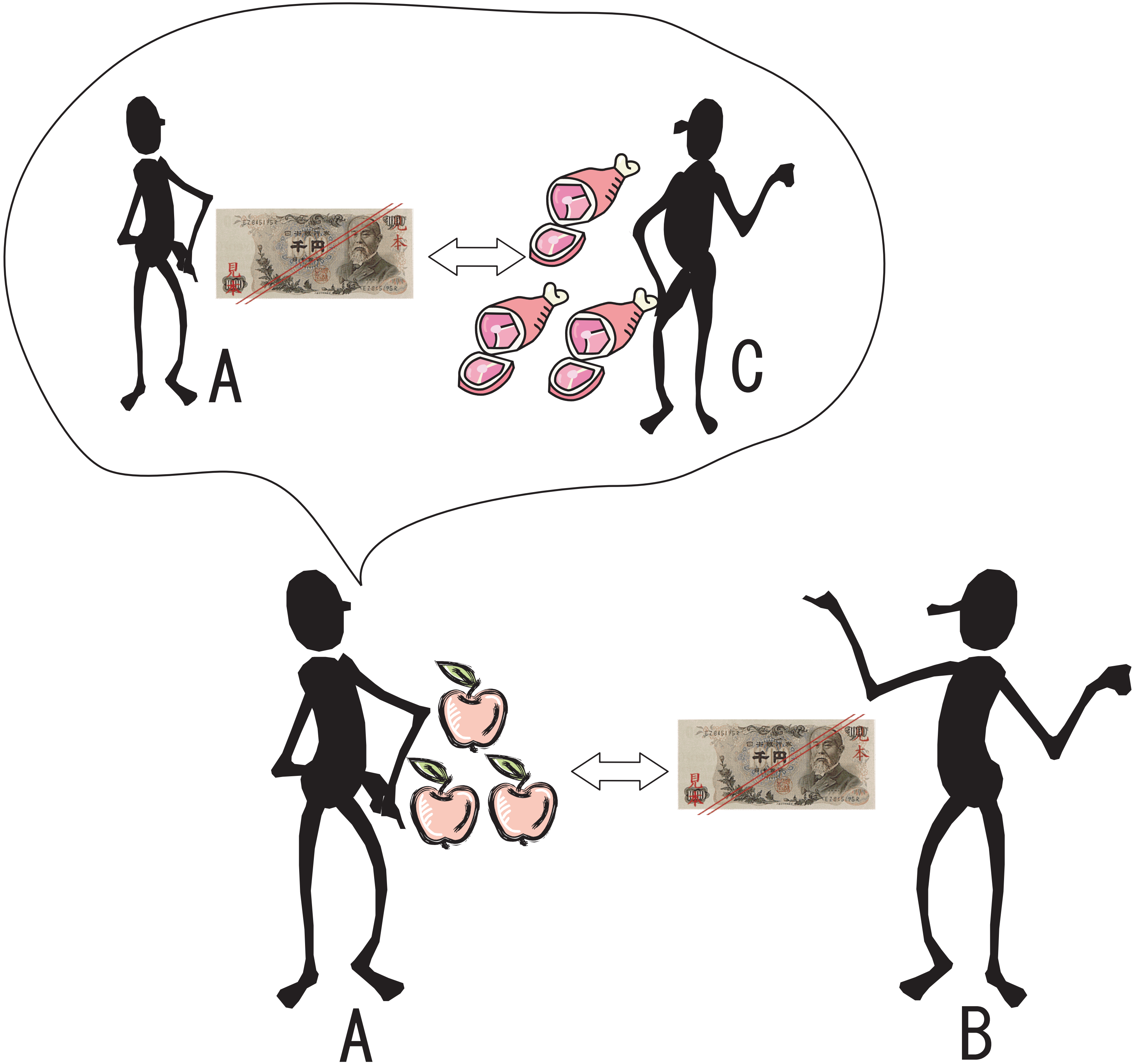
Indeterminacy of Monetary Equilibria

In Iwai, Kiyotaki, and Wright's models, if everyone believes that money has value, it indeed has value in equilibria.

In Green, Zhou, Kamiya, and Shimizu's models, if everyone believes that money has big value, it indeed has big value in equilibria, and if everyone believes that money has small value, it indeed has small value in equilibria.

Note that indeterminacy occurs in **matching models** with **divisible modey**.

Even when money is divisible, indeterminacy does not occur in **centralized Walrasian markets** with **cash-in-advance constraints.**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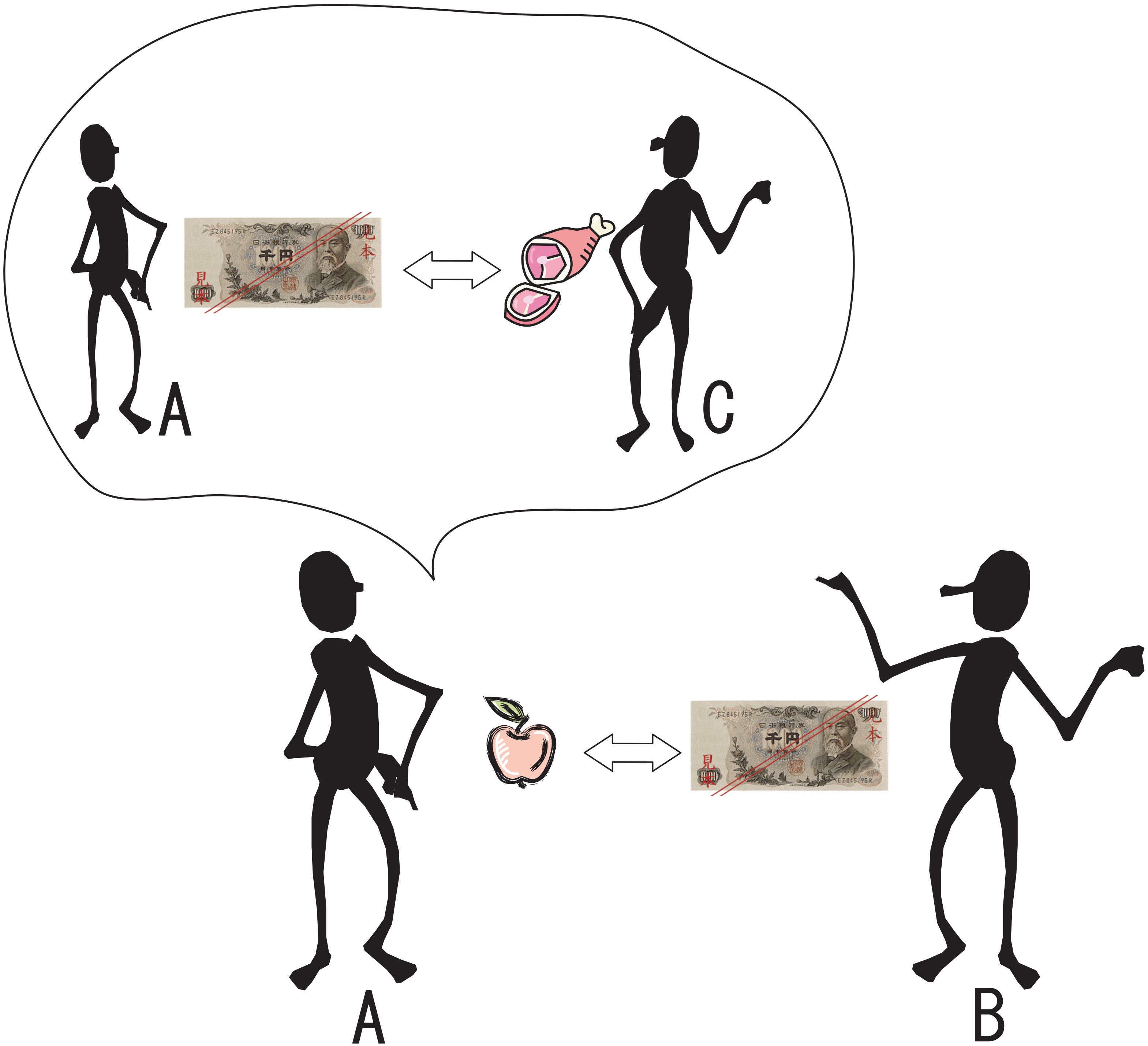
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Indeterminacy of Monetary Equilibria

The logic of indeterminacy:

$\boldsymbol{M} > \mathbf{0}$: The amount of fiat money.
$\boldsymbol{p} > \mathbf{0}$: a price of good

Indeterminacy of Monetary Equilibria

Example 1:

$\{0, p, 2p\}$: the set of money holdings
$\{A, B, C, D\}$:the set of agents. (In the next example, I consider the case of a continuum of agentsj

- The money holding of A is p
- The money holding of B is $2p$
- The money holding of C is 0
- The money holding of D is $2p$

Indeterminacy of Monetary Equilibria

At time t,

1. (i) A meets B, and (ii) C meets D
2. Then (i) B buys goods from A (ii) D buys goods from C

(the price is p.)

- The money holding of A: $p \to 2p$
- The money holding of B: $2p \to p$
- The money holding of C: $0 \to p$
- The money holding of D: $2p \to p$

Indeterminacy of Monetary Equilibria

$\mathcal{O}_n$: The set of agents who have ***np*** **before** the trades.
$\mathcal{I}_n$: The set of agents who have ***np*** **after** the trades.

- $A \in \mathcal{O}_1, A \in \mathcal{I}_2$
- $B \in \mathcal{O}_2, B \in \mathcal{I}_1$
- $C \in \mathcal{O}_0, C \in \mathcal{I}_1$
- $D \in \mathcal{O}_2, D \in \mathcal{I}_1$

Then

$$(*) \quad \sum_{n=0}^{2} \#\mathcal{O}_n = \sum_{n=0}^{2} \#\mathcal{I}_n \quad \text{(identityj}$$

holds.

Moreover

(i) $\boldsymbol{B}$ buys goods from $\boldsymbol{A}$ $\rightarrow$

The money holding of $\boldsymbol{A}$ before the trade
$+$
The money holding of $\boldsymbol{B}$ before the trade
$=$
The money holding of $\boldsymbol{A}$ after the trade
$+$
The money holding of $\boldsymbol{B}$ after the trade

(ii) $\boldsymbol{D}$ buys goods from $\boldsymbol{C}$ $\rightarrow$

The money holding of $\boldsymbol{C}$ before the trade
$+$
The money holding of $\boldsymbol{D}$ before the trade
$=$
The money holding of $\boldsymbol{C}$ after the trade
$+$
The money holding of $\boldsymbol{D}$ after the trade

Then

$$(**) \quad \sum_{n=0}^{2} np\#\mathcal{O}_n = \sum_{n=0}^{2} np\#\mathcal{I}_n \quad \text{(identity)}$$

$$p\#\mathcal{O}_1 + 2p\#\mathcal{O}_2 = p\#\mathcal{I}_1 + 2p\#\mathcal{I}_2$$

In a stationary equilibrium, the money holdings distribution should be stationary.

The stationarity condition:

$$\#\mathcal{I}_0 = \#\mathcal{O}_0$$

$$\#\mathcal{I}_1 = \#\mathcal{O}_1$$

$$\#\mathcal{I}_2 = \#\mathcal{O}_2$$

By $(*)$, $\#\mathcal{I}_1 = \#\mathcal{O}_1$ and $\#\mathcal{I}_2 = \#\mathcal{O}_2$ imply $\#\mathcal{I}_0 = \#\mathcal{O}_0$.

By $(**)$, $\#\mathcal{I}_2 = \#\mathcal{O}_2$ implies $\#\mathcal{I}_1 = \#\mathcal{O}_1$.

Thus $\#\mathcal{I}_2 = \#\mathcal{O}_2$ implies $\#\mathcal{I}_1 = \#\mathcal{O}_1$ and $\#\mathcal{I}_0 = \#\mathcal{O}_0$.

Example 2:

$\{0, p, 2p\}$: The set of money holdings
$[0, 1]$: The set of agents
O_n: The measure of agents with np before trade.
I_n: The measure of agents with np after trade.

Then

$$(*)\quad \sum_{n=0}^{2} O_n = \sum_{n=0}^{2} I_n \text{ (identity)}$$

$$(**)\quad \sum_{n=0}^{2} npO_n = \sum_{n=0}^{2} npI_n \text{ (identity)}$$

$$I_0 = O_0$$
$$I_1 = O_1$$
$$I_2 = O_2$$

By $(*)$, $I_1 = O_1$ and $I_2 = O_2$ imply $I_0 = O_0$.

By $(**)$, $I_2 = O_2$ implies $I_1 = O_1$.

Then $I_2 = O_2$ implies $I_1 = O_1$ and $I_0 = O_0$.

h_n: The measure of agents with np
$h = (h_0, h_1, h_2)$
Suppose O_n and I_n are functions of h.
The stationarity condition:

$$I_0 = O_0$$
$$I_1 = O_1$$
$$I_2 = O_2$$
$$h_0 + h_1 + h_2 = 1$$

The number of independent equations=2
The number of variables=3
The degree of freedom =1

Indeterminacy of Monetary Equilibria

The condition for a stationary equilibrium:

$$
\begin{aligned}
I_0 &= O_0 \\
I_1 &= O_1 \\
I_2 &= O_2 \\
h_0 + h_1 + h_2 &= 1 \\
\sum_{n=0}^{2} p n h_n &= M
\end{aligned}
$$

(The other equations, e.g. Bellman equations)

In the other equations, the number of equations is typically equal to the number of variables.
$\rightarrow$ the degree of freedom is equal to one.

Indeterminacy of Monetary Equilibria

How to understand the indeterminacy?

The general consensus of indeterminacy is either

1. due to the absence of some important equation, or
2. equilibria in the real world economy are intrinsically fragile. Thus we need a policy which induces a determinate and efficient equilibrium.
3. A specific equilibrium is selected as a focal point. (Experiment)

The first approach: Lagos and Wright (2005)
Some goods are traded in centralized Walrasian markets.

The second approach: Kamiya and Shimizu (2007)
A certain tax-subsidy scheme induces a determinate and efficient equilibrium.

Literature

- Duffy and Ochs (1999, 2002): the 1st gereneration model, i.e., money is indivisible
- Duffy and Puzzello (2014): Lagos and Wright model) degenerate money holdings distributions
- This paper: a continuum of non-degenerate money holdings distributions

Results

- In some treatment, there is a tendency to converge to the most efficient stationary equilibrium
- However, as a whole, there is some degree of indeterminacy
- There are systematic deviations from our target equilibria
  - Many subjects avoid spending all their money holdings
  - Some subjects become inactive when they are sellers as the session lasts longer

- Baseline model = a variant of Zhou (1999)'s model
- Time: $t = 1, 2, \dots$
- Each agent can produce one unit of goods in each period
- She cannot consume the good she produces by herself and can consume the goods the other agents produce
- Goods: indivisible and perishable

Money and Matching

- Money is divisible and each agent can hold any amount of money
- Each agent can observe the current money holdings distribution in the beginning of each period
- Pairwise random matchings take place in each period
- In each matching, one agent becomes a seller and the other becomes a buyer (random assignment)
- The bargaining protocol: the seller's take-it-or-leave-it offer
- The seller cannot observe the buyer's money holding

Parameters

- u: the utility of consumption
- c: the cost of production
- $u > c > 0$
- In the end of each period, the economy ends with probability $1 - \delta$, while it goes to the next period with probability δ
- Agents do not discount future payoffs

Single Price Equilibrium

We focus on *single price equilibria (SPE)* as follows:

- I: total number of agents
- I_n: number of agents holding np amount of money
- $(I_0/I, I_1/I)$: stationary money holdings distribution
- p is determined by $M = pI_1$ where M is the nominal stock of money
- A seller offers a price p if her current money holding is $\eta < p$, and otherwise she offers a price that cannot be accepted by any buyer
- A buyer accept a price offer p whenever her money holding is $\eta \geq p$

Single Price Equilibrium (cont'd)

In a nutshell, on the equilibrium path of SPE with $\boldsymbol{p}$,

- transaction only occurs between a seller with $\mathbf{0}$ and a buyer with $\boldsymbol{p}$,
- $\#$ of potential buyers $= \#$ of money holders, and
- an individual money holding stochastically alternates between $\mathbf{0}$ and $\boldsymbol{p}$

Multiple SPEs

- $I = 6$
- $\delta = 0.9$
- $c = 10$
- $M = 600$

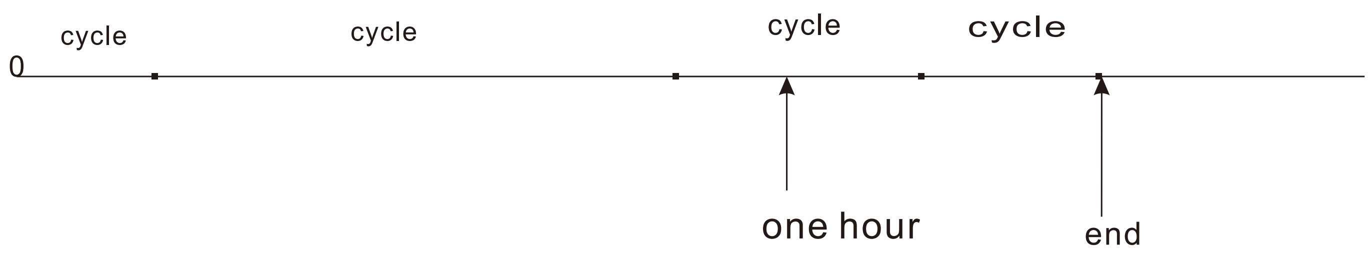
1. $u = 14$
   $\Rightarrow$ SPE: (p , # of money holders) = (200,3), (300,2), (600,1)
2. $u = 20$
   $\Rightarrow$ SPE: (p , # of money holders) = (150,4), (200,3)

Variation: SPE with Residuals

- Suppose $(200, 200, 200, 0, 0, 0)$ and $p = 200$ constitute an SPE
- $(190, 190, 190, 10, 10, 10)$ and $p = 180$ also consitute an SPE as long as the discount factor is not so large
- Residual=10: a small portion of money that is not used in transaction and has no value by itself

Experimental Design

- Each session consists of several sequences
- Each sequence consists of an indefinite number of periods
- The experiments were conducted at Kansai University on January 2015–July 2016
- In each session, 24 or 18 subjects interacted through z-Tree software (Fischbacher 2007)
- Total points = 300 (showup fee) + points acquired in the session
- 1 point = 10 JPY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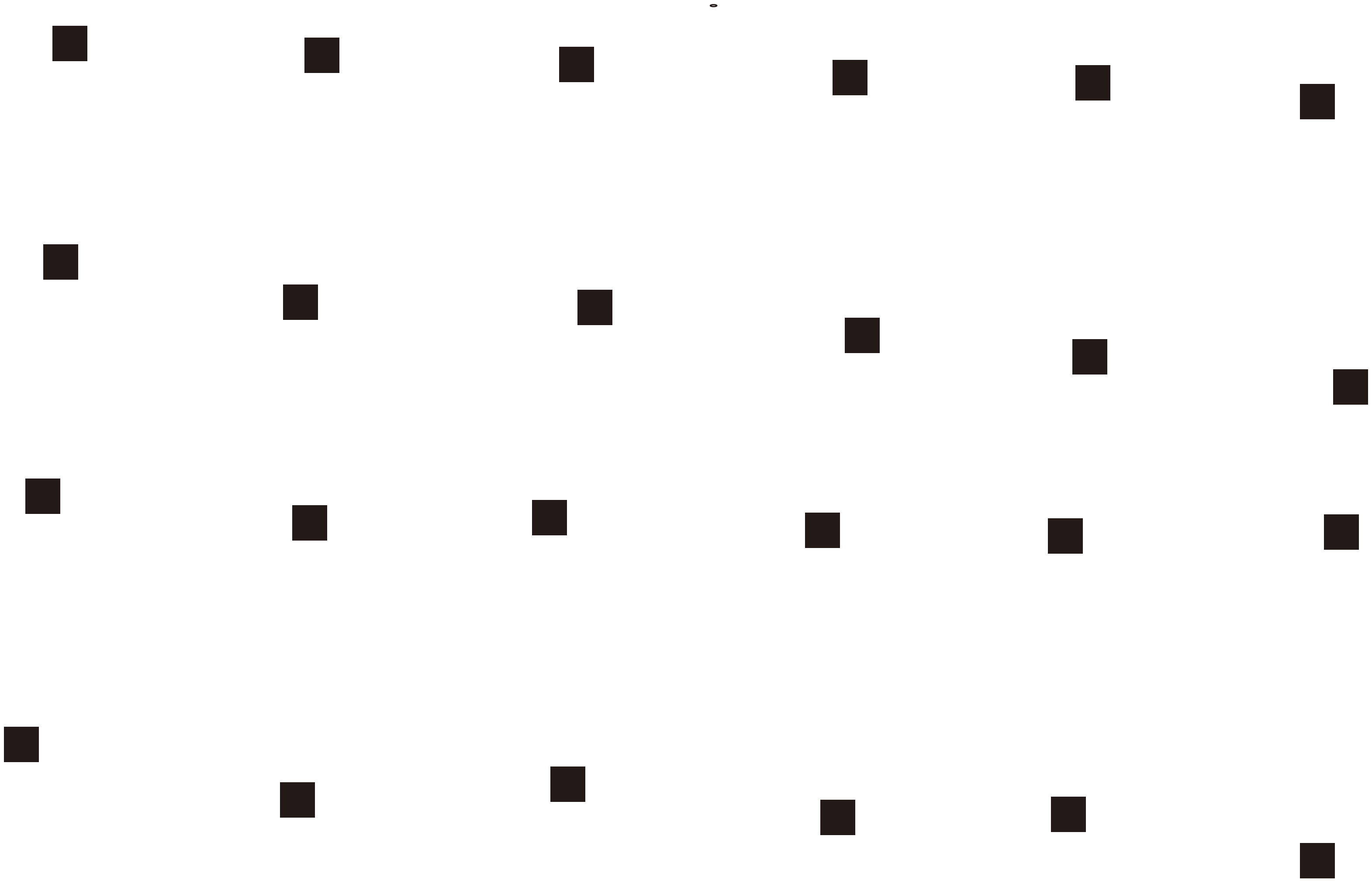
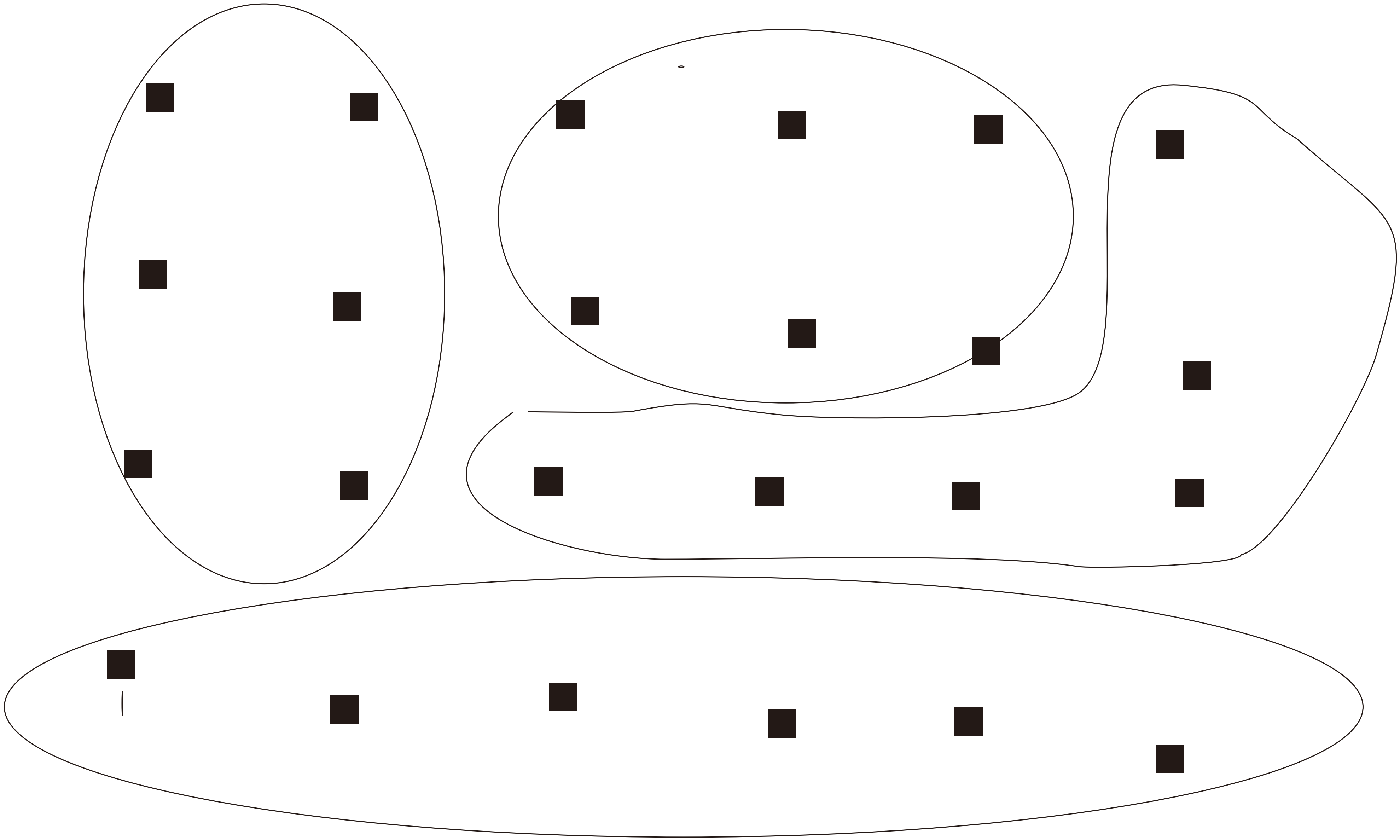
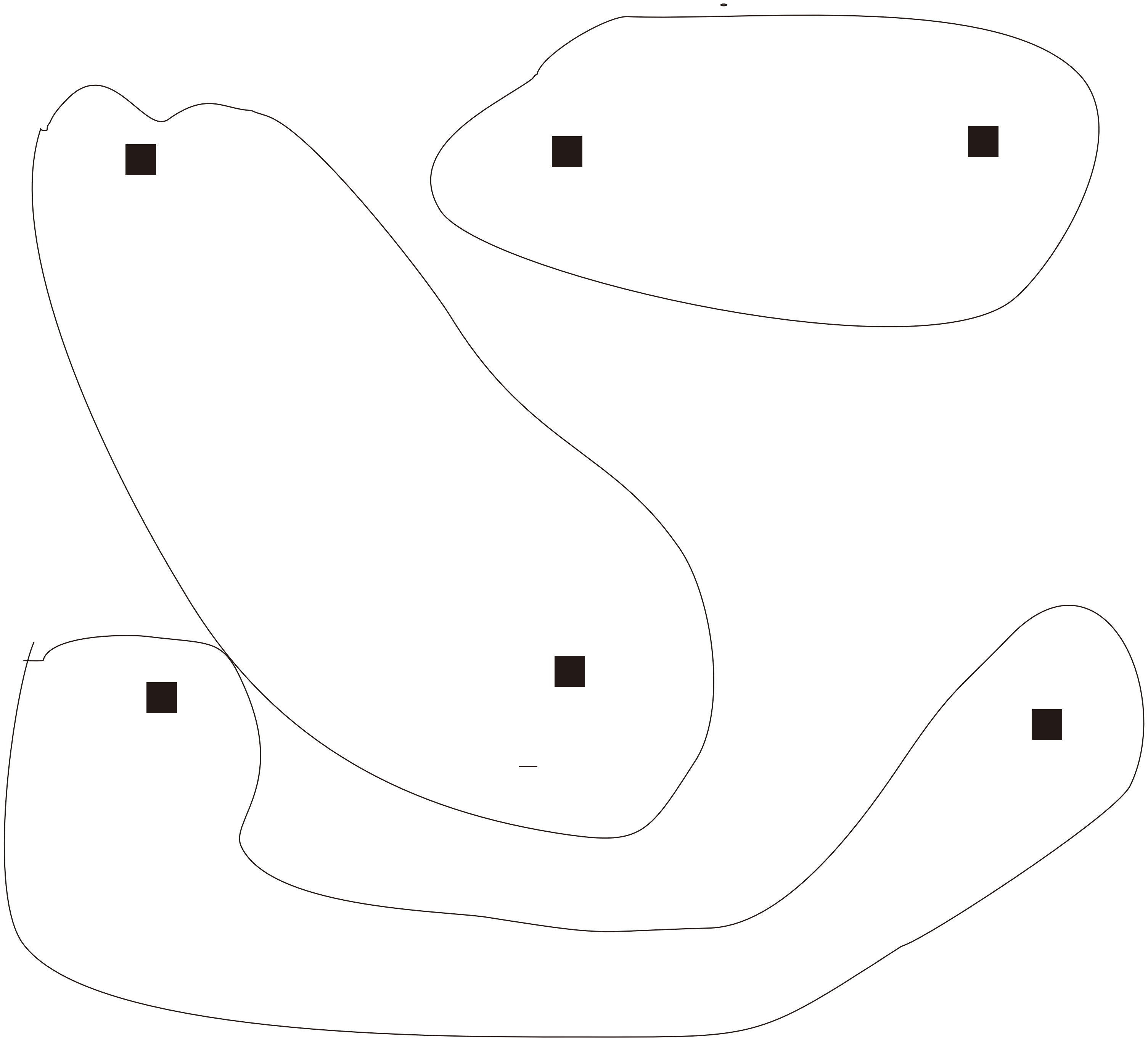
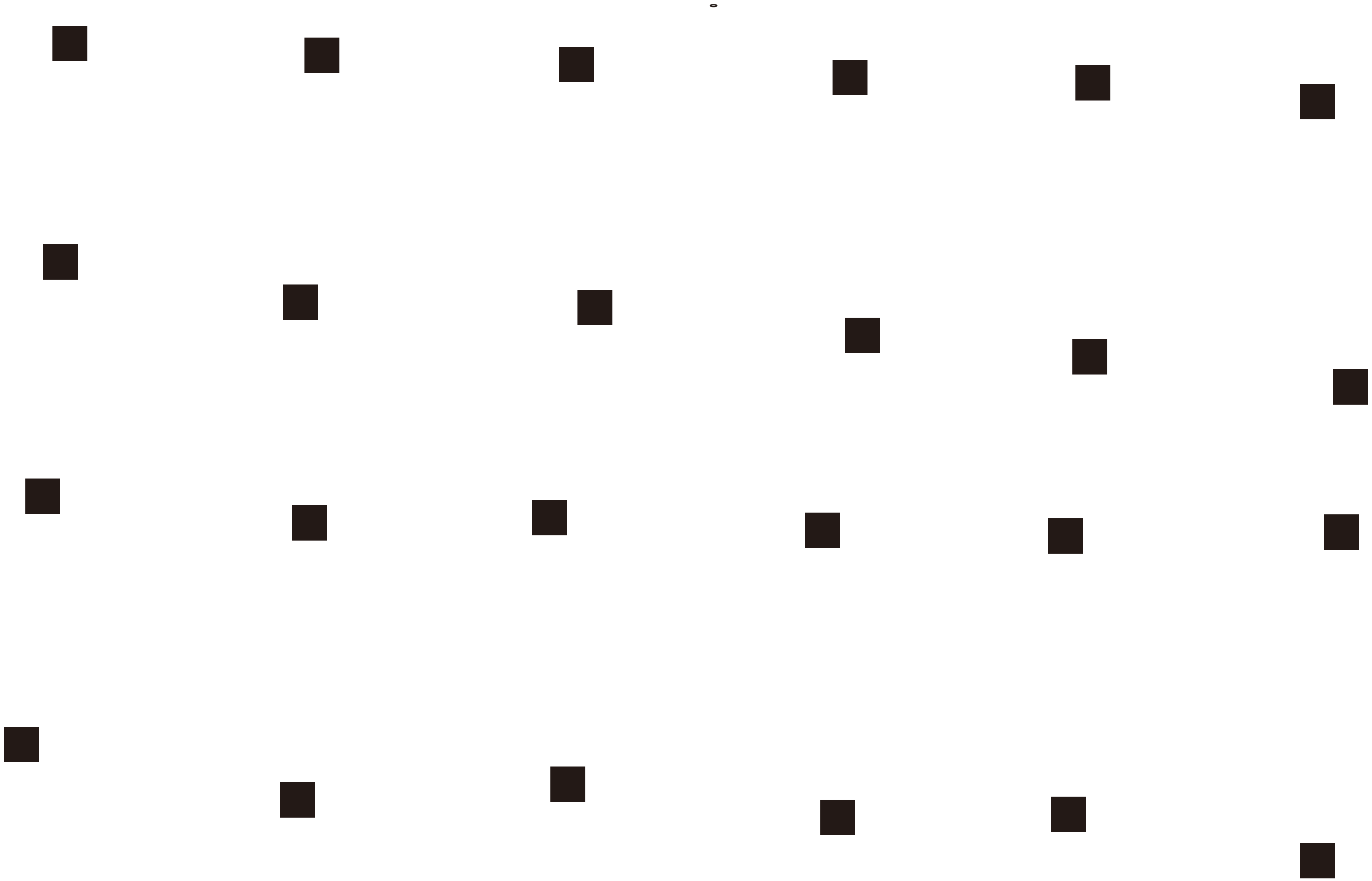
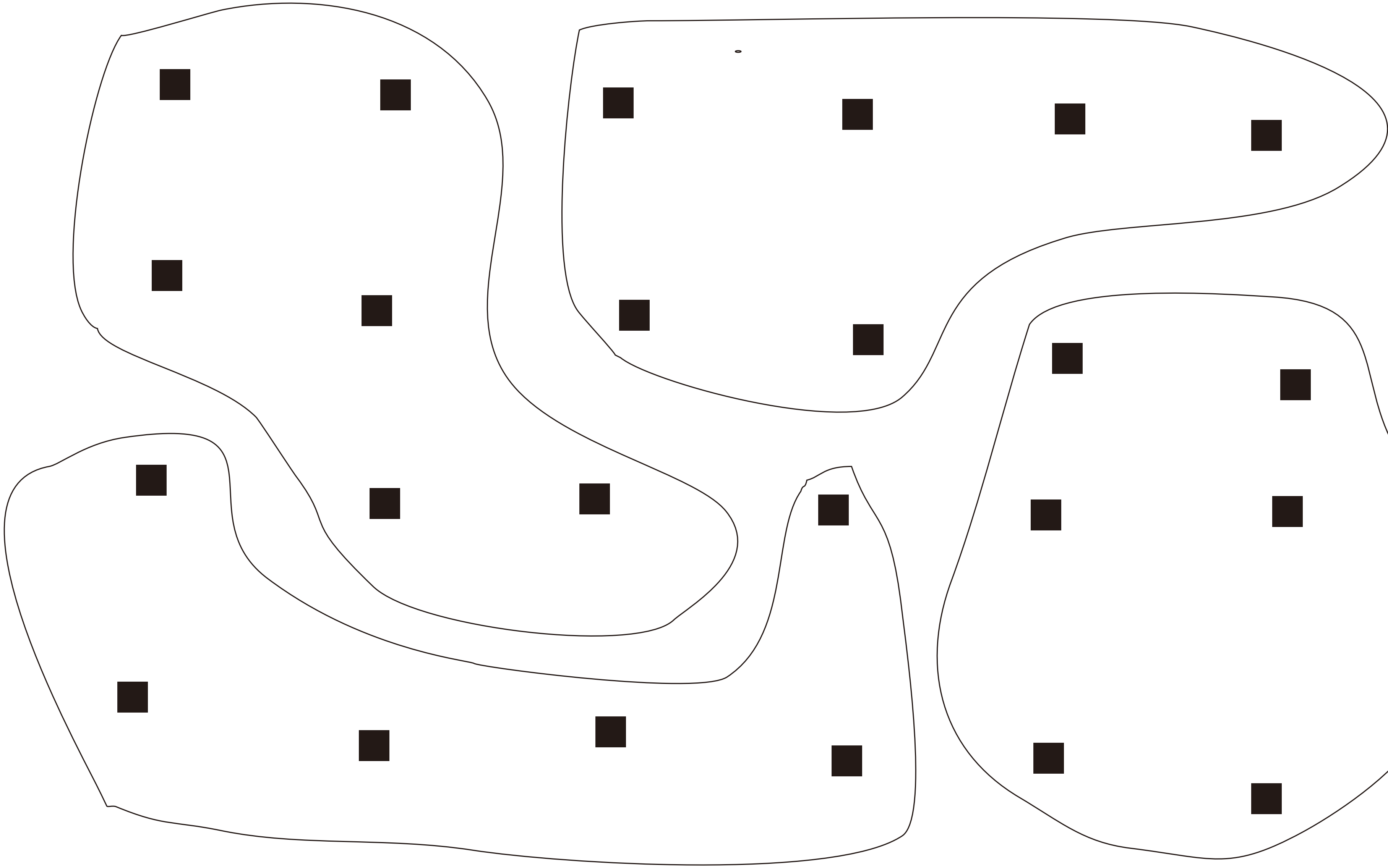
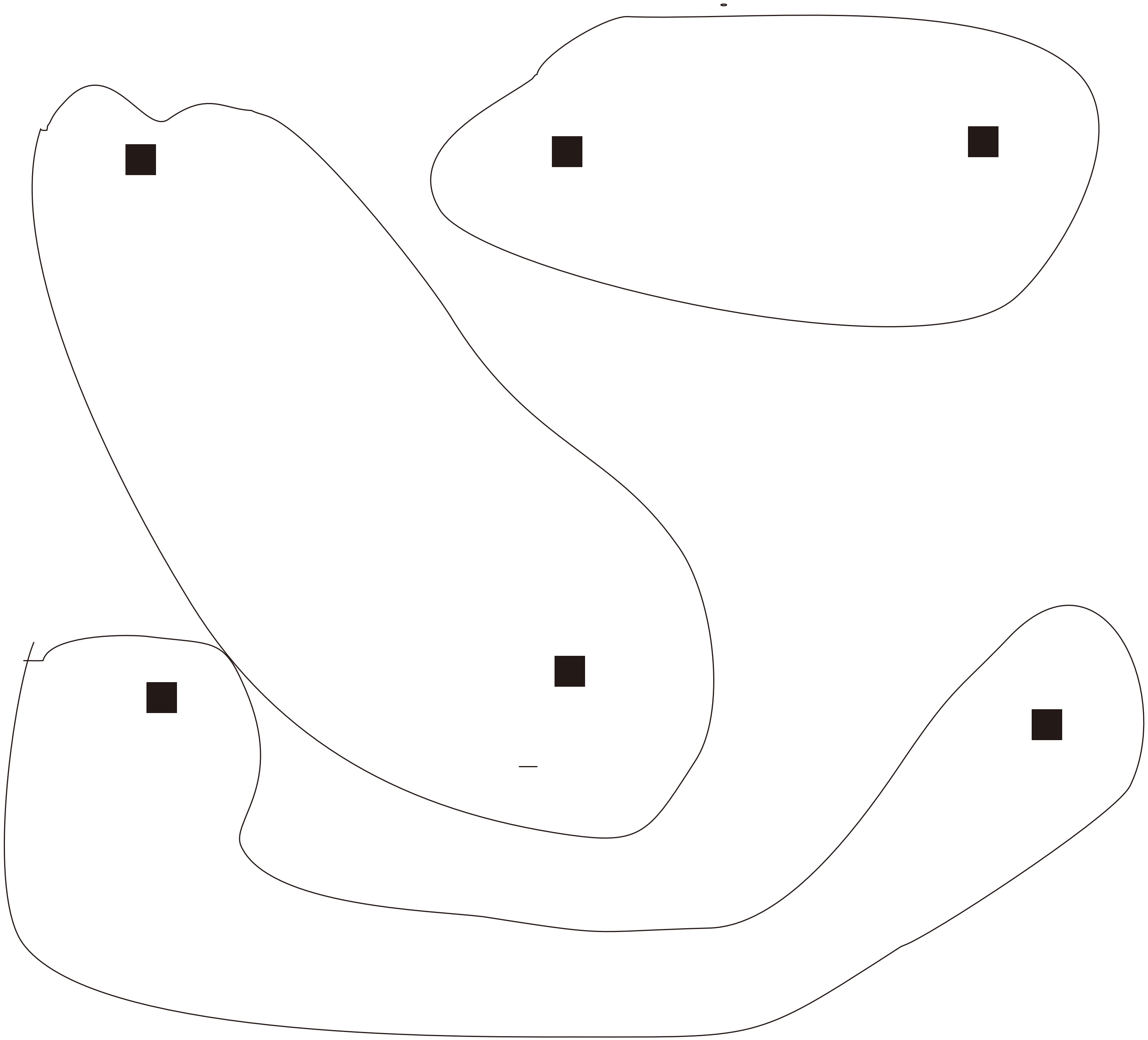












Summary of Experimental Sessions

Session No., Treatment	Subjects	Total Periods	Sequences	Ave. Duration
1: u=14, dis200, ShowPrice=0	6×4	44	2	22
2: u=14, dis200, ShowPrice=0	6×4	76	4	19
3: u=14, dis100, ShowPrice=0	6×4	45	9	5
4: u=14, dis100, ShowPrice=0	6×4	48	8	6
5: u=14, dis300, ShowPrice=0	6×4	50	10	5
6: u=14, dis300, ShowPrice=0	6×4	55	7	7.9
7: u=14, dis100, ShowPrice=0	6×4	55	8	6.9
8: u=14, dis200, ShowPrice=0	6×4	53	7	7.6
9: u=14, dis300, ShowPrice=0	6×4	55	2	27.5
10: u=14, dis200, ShowPrice=1	6×4	49	7	7
11: u=14, dis300, ShowPrice=1	6×4	55	10	5.5
12: u=14, dis300, ShowPrice=1	6×4	54	7	7.7
13: u=14, dis100, ShowPrice=1	6×4	53	4	13.25
14: u=14, dis100, ShowPrice=1	6×4	57	6	9.5
15: u=14, dis200, ShowPrice=1	6×4	49	6	8.2
16: u=20, dis200, ShowPrice=1	6×3	44	4	11
17: u=20, dis200, ShowPrice=1	6×4	64	8	8
18: u=20, dis100, ShowPrice=1	6×3	46	2	23
19: u=20, dis100, ShowPrice=1	6×4	48	4	12
20: u=20, dis150, ShowPrice=1	6×4	48	6	8
21: u=20, dis150, ShowPrice=1	6×4	66	7	9.4

Convergence of Offer Price

- Samples: **SequenceLength** ≥ 21
- Data: **PeriodInSequence** ≥ 6
- $\boldsymbol{P_t}$:average serious offer price within the group
  - Serious offer=offering a price that can be accepted by some member
  - No-sale offer=offering a price that cannot be accepted by any member

Convergence of Offer Price (cont'd)

- $P_t - P_{t-1} = \alpha + \beta P_{t-1} + \varepsilon_t$
- Dicky-Fuller test: $H_0 : \beta = 0$
- Non-convergence: groups in which the null hypothesis is not rejected with significance level of 10%
- **LimitPrice**: estimated value of $-\frac{\alpha}{\beta}$
- **LB**: lower bound of 95% confidence interval of **LimitPrice**
  $\doteqdot$ cautious proxy of limit price
- **Rich**: average number of subjects whose money holdings are above **LB** within the last 5 periods
  $\doteqdot$ number of hypothetical buyers

Convergence of Offer Price (cont'd)

- **Residual**: {M–**LimitPrice*Rich** } / 6
- Inconsistent groups: **Residual** $\geq$ **LimitPrice**
- **Class-N**: convergent and consistent groups whose **Rich** is the closest to ***N***
  $\doteqdot$ groups converging to SPE with ***N*** buyers

Result 2: Residuals

All convergent and consistent groups have a significant amount of residuals

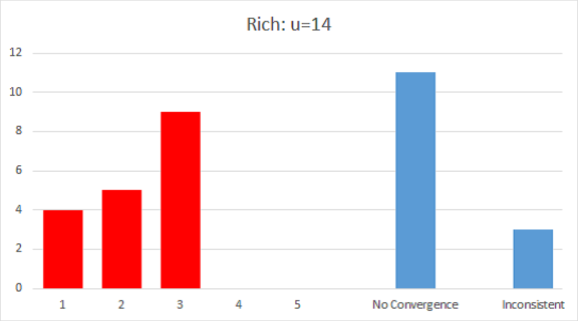
	Residual
Mean	46.86
Min	15.57
Max	77.15

Random Effects, Probit Analysis on Buyer's Acceptance

	(1)	(2)	(3)	(4)
Period	0.0398***	0.0365***	0.0385***	0.0354***
	(0.00242)	(0.00228)	(0.00310)	(0.00295)
Offer Price	-0.0155***		-0.0155***	
	(0.000668)		(0.000867)	
Money Holding	0.00776***		0.00806***	
	(0.000544)		(0.000704)	
Remaining Amount		-0.00120		-0.000451
		(0.000813)		(0.00106)
Remaining Proportion		3.076***		2.837***
		(0.197)		(0.251)
Holt-Laury Score			-0.0598*	-0.0685**
			(0.0336)	(0.0343)
Constant	0.487***	-0.820***	0.737***	-0.415*
	(0.0802)	(0.0825)	(0.239)	(0.248)
Observations	4,349	4,322	2,523	2,511

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

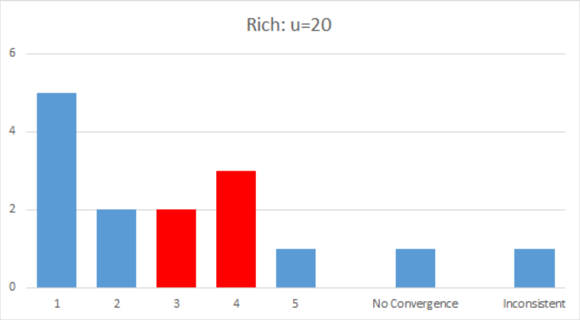
Number of Hypothetical Buyers: $\mathbf{u = 14}$



Result 4: Most Efficient Equlibrium Is Most Likely in $u = 14$

In the case of $\boldsymbol{u = 14}$, the sequences of offer prices are most likely to converge to the most efficient stationary equilibrium. most likely to converge to

Number of Hypothetical Buyers: $\mathbf{u = 20}$



Result 5: More Likely to Converge in $u = 20$

The frequency of non-convergent groups is much less in the case of $\boldsymbol{u = 20}$ than in the case of $\boldsymbol{u = 14}$

Interpretation of Result 5: Higher Buyer's Acceptance Rate in $u = 20$

	Acceptance Rate
u=14	0.34
u=20	0.41

- Wilcoxon-Mann-Whitney test: the null hypothesis is rejected (P=0.0000)
- Conjecture: More acceptance rate $\Rightarrow$ more likely to converge

Result 6: Fewer Buyers in $u = 20$

In the case of $\boldsymbol{u = 20}$, the number of hypothetical buyers (**Rich**) is less than that in SPEs

Interpretation of Result 6: Inactive Sellers

- **InactiveSellerRate**: the rate of no-sale offers among all offers when seller's current money holdings are less than **LB** within the last 10 periods
- **InactiveSellerRate** is above 0.4 among 5 out of 7 samples with fewer buyers than the theory predicts

Random Effects, Probit Analysis on Seller's No-Sale Offer

- Samples: sellers whose money holdings are the least within the groups they are belonging to

	(1)	(2)	(3)
Period	0.0470***	0.0465***	0.0494***
	(0.00268)	(0.00568)	(0.00376)
AcquiredPoints		0.00226	
		(0.00353)	
Holt-Laury score			0.00458
			(0.103)
Constant	-3.441***	-3.762***	-3.983***
	(0.182)	(1.142)	(0.728)
Observations	4,073	1,386	2,102

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Conclusion

- We investigate the convergence of sellers' offer prices in a moentary serach model in which there are multiple stationary monetary equilibria
- In some treatment, there is a tendency to converge to the most efficient equilibrium
- However, as a whole, there is some degree of indeterminacy
- There are systematic deviations from typical SPEs
    - Many subjects avoid spending all their money holdings and keep some residuals with them
    - Some subjects become inactive when they are assigned the roles of sellers as the session lasts longer

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