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Arrovian Social Choice with Non-Welfare Attributes

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1　序論:論文の主題

- 厚生情報と非厚生情報に基づいたアロウ型社会的選択を考える.
- 厚生情報のみ:アロウの不可能性定理
- 非厚生情報をもカウントすることによって，この結果がどのように変わるか.

2　先行研究

Fleurbaey M (2003) On the informational basis of social choice, Social Choice and Welfare 21: pp347–384

Sen AK (1969) The impossibility of the Paretian liberal, Journal of Political Economy 78 pp152-157

Sen AK (1970) Collective Choice and Social Welfare Holden-Day SanFrancisco

Sen AK　(1979) Utilitarianism and Welfarism,　The Journal of Philosophy 76 pp463–489

3　序論:リベラルパラドクス（Sen 1969）

- **結婚のパラドックス**（Gibbard 1974）

$$\text{アンジェリーナ} \quad : \quad E \quad \overbrace{J \quad O}$$

$$\text{エドウィン} \quad : \quad \overbrace{O \quad E} \quad J$$

E :**アンジェリーナとエドウィンの結婚**

J :**アンジェリーナと判事の結婚**

O :**アンジェリーナもエドウィンも独身**

- **二人の自由を認めるとjが帰結．しかしこれはパレート最適ではない．**

4　リベラルパラドックスと厚生主義批判

- 厚生主義 (Welfarism)：社会的選択は厚生情報のみに基づいて決めるべき．
- Sen はリベラルパラドックスの議論から出発し，この立場を強く批判．

5　リベラルパラドックスと厚生主義批判

- 例を読みかえる．二人が一緒に
  E : 洋風料理レストランに行く
  J : 日本料理レストランに行く
  O : 家で食べる．
- 厚生主義の立場に立てば結婚問題とディナー問題は同じ解を持たねばならない．
- 結婚問題では「結婚の自由・権利」という非厚生情報があり，社会的選択においてはこれをカウントしなければならない．厚生主義は否定されざるを得ない．

6　情報的基礎:道徳哲学

- 功利主義:ベンサム，J.S. ミル（古典的功利主義），ヘア（現代功利主義）
- 自由の尊重:ノージック，カント，ロールズ．ノージックは平等への配慮なし．カント，ロールズは平等に高い価値を置く．
- 卓越主義:アリストテレス，セン，サンデル．

7　情報的基礎:功利主義

- ベンサム　厚生情報のみ．各自の幸福（効用）の集計和のみを使う．
- J.S. ミル　ベンサムと同じ厚生情報を使うが，人格の完成（自由の尊重）という非厚生情報もコッソリ入れる．
- ヘア　厚生情報と非厚生情報．批判レベルでは厚生情報のみ．直観レベルでは非厚生情報も使う．

8　情報的基礎:自由の尊重

- ノージック　自由の無条件の尊重，非厚生情報のみ．
- カント　　　非厚生情報のみ（人格の尊重・完成），カントの言う自由＝心が解放された状態
- ロールズ　　両方使う．社会的基本財の分配（自由と権利，権力と地位，富と所得）

9　情報的基礎:卓越主義

- **アリストテレス，セン，サンデル　3人とも非厚生情報（美徳）のみ**

  - **ただし美徳の増進のためのインセンティブとして厚生情報への配慮も忘れていない．**

10　情報的基礎:経済学

- **厚生情報のみ**:**ワルラスルールの公理化** (Nagahisa and Suh 1995).
  市場を通した資源配分は初期資産と同程度以上の効用を保証し（IR)，**パレート最適であり** (PE)，**限界代替率だけで決まる**(LI).
- **再分配に関する様々な基準**(No Envy，**ロールズ的格差基準など）はあるが，その情報的基礎は選好や効用のみ．**

11　情報的基礎:現実の経済問題

- 現実の政策問題では非厚生情報も使っている．
- 社会財の存在（医療・教育など）．市場で供給できるが市場のみに任せるわけにはいかない．
- コカ・コーラは市場だけで売り買いしてもいいが，医療と教育は市場だけでというわけにはいかない．
- この問題は格差問題には還元できない．コーラの所得ごと消費格差と医療・教育所得ごと消費格差は違う．

12 Model

- $N = \{1, 2, ..., n\}$: the finite set of persons with at least two.
- X: the finite set of social states with at least three.
- $\succcurlyeq_i$: the preference of person i, complete and transitive on X.
- $P(X)$: the set of all preferences.
- $\succcurlyeq = (\succcurlyeq_1, ..., \succcurlyeq_n)$; a profile.
- $P(X)^n$: the set of profiles

13 Model

- A social choice rule F :

$$\succsim = (\succsim_1, ..., \succsim_n) \in P(X)^n \stackrel{F}{\longrightarrow} \succsim_F$$

$\succsim_F$: a social preference, a complete binary relation on X.

14 Non-Welfare Attributes

- A social state $x \in X$ is characterized by welfare and non-welfare attributes.
- Given a profile, the welfare attribute of x are the profile itself and all the concepts derived from profiles such as utilities of x, the Borda numbers of x and so on.
- On the other hand non-welfare attributes are intrinsic to x independently from profiles.

15 Non-Welfare Attributes

- Let a non-welfare attribute be given.
- All social states are classified into subgroups in which each member is thought of as identical from the viewpoint of the non-welfare attribute.

16 Non-Welfare Attributes

- X has a partition $\{X_\lambda\}_{\lambda \in \Lambda}$, i.e., $X = \bigcup_{\lambda \in \Lambda} X_\lambda$ and $X_\lambda \cap X_{\lambda'} = \emptyset$ for all $\lambda \neq \lambda'$.
- If $x, y \in X_\lambda$, we cannot distinguish between x and y from the viewpoint of the non-welfare attribute.
- We call X_λ an attribute set. We assume that there exist at least two attribute sets.

17 Example: Lady Chatterley's Lover (Sen 1969)

$X = \{r_{AB}, r_A, r_B, r_0\}$.

The non-welfare attribute: Read or not, a kind of morality

The attribute sets:

either $\{r_{AB}\}, \{r_A, r_B\}, \{r_0\}$

or $\{r_{AB}, r_A, r_B\}, \{r_0\}$.

18 Example: Marriage (Gibbard 1974)

$X = \{E, J, O\}$.

The non-welfare attribute: Marriage.

The attribute sets are $\{E, J\}$ and$\{O\}$ whereas $\{E, J, O\}$ when dinner is concerned.

19 Example: Mac or Windows

$X = \{(m, m), (m, w), (w, m), (w, w)\}$.

The non-welfare attribute: Corporation.

The attribute sets:

$\{(m, m), (w, w)\}, \{(m, w), (w, m)\}$.

Example: Environment

Building a commercial complex (C) or protecting natural environment (E)

$$X = \{C, E\} \times \prod_{i=1}^{n} X_i.$$

The non-welfare attribute: Environment.

The attribute sets:

$$\{C\} \times \prod_{i=1}^{n} X_i, \{E\} \times \prod_{i=1}^{n} X_i.$$

21 Conditional Full Rationality (CFR)

- A rule F satisfies Conditional Full Rationality (CFR)if $\forall \succsim \in P(X)^n$, $\forall X_\lambda, X_{\lambda'}, \lambda \neq \lambda'$ and $\forall \{x, y, z\} \subset X_\lambda \cup X_{\lambda'}$, $x \succsim_F y \succsim_F z$ implies $x \succsim_F z$.
- Note that transitivity of $\succsim_F$ does not always hold on $\{x, y, z\}$ if each of the three belongs to a different attribute set.
- Full Rationality (FR): $\forall \succsim \in P(X)^n$, $\forall x, y, z \in X$, $x \succsim_F y \succsim_F z$ implies $x \succsim_F z$.

 There are four cases for CFR.

	X_λ	$X_{\lambda'}$
Case 0	x, y, z	
Case 0		x, y, z
Case 1	x, y	z
Case 2	x	y, z
Case 3	x, z	y

$x \succsim_F y \succsim_F z$ implies $x \succsim_F z$.

23 Case 1(Case 2)

$$x \succsim_F y \succsim_F z \Longrightarrow x \succsim_F z.$$

x: We are Christian with a piece of bread

y: We are Christian with no bread

z: We are not religious with bread as much as we like

Case 1(Case 2)

$$x \succsim_F y \succsim_F z \Longrightarrow x \succsim_F z.$$

x	y	z
Christian	Christian	non-Christian
a piece of bread	no bread	as much as you like

Case 3

$$x \succcurlyeq_F y \succcurlyeq_F z \Longrightarrow x \succcurlyeq_F z.$$

x: We are Christian with a piece of bread

y: We are not religious with bread as much as we like

z: We are Christian with no bread

Case 3

$$x \succcurlyeq_F y \succcurlyeq_F z \Longrightarrow x \succcurlyeq_F z.$$

x	y	z
Christian	non-Christian	Christian
a piece of bread	as much as you like	no bread

27 transitivity of $\succsim_F$ does not always hold

x:We are Christian and smokers, and same-sex marriage is not legalized

y:We are non-Christian and nonsmokers, and same-sex marriage is not legalized

z:We are are non-Christian and smokers, and same-sex marriage is legalized

28 transitivity of $\succsim_F$ does not always hold

$$x \succsim_F y \succsim_F z \Longrightarrow x \succsim_F z?$$

	x	y	z
religion(Christian)	yes	no	no
health(Nonsmoking)	no	yes	no
sex(S.S.Marriage)	no	no	yes

29 Binary Independence (BI),

- A rule F satisfies Binary Independence (BI) if $\forall \succsim, \succsim' \in P(X)^n$ and $\forall x, y \in X$, if $\succsim_i \cap \{x, y\}^2 = \succsim'_i \cap \{x, y\}^2 \ \forall i \in N$, then $\succsim_F \cap \{x, y\}^2 = \succsim'_F \cap \{x, y\}^2$.

Binary Pareto (BP) and Indifference Pareto (IP)

- A rule F satisfies Binary Pareto (BP) if $\forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in X$, if $x \succ_i y \ \forall i \in N$, then $x \succ_F y$.
- A rule F satisfies Indifference Pareto (IP) if $\forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in X$, if $x \sim_i y \ \forall i \in N$, then $x \sim_F y$.

Dictatorial Powers

- Person i is decisive for (x, y) if $\forall \succcurlyeq \in P(X)^n$, $x \succ_i y$ implies $x \succ_F y$.
- Person i is dictator on $Y \subset X$ if he is decisive for any pair in $Y \times Y$.
- Person i is dictator if he is dictator on X.
- Person i is complete dictator on $Y \subset X$ if for any $x, y \in Y$, $x \succcurlyeq_i y \Longleftrightarrow x \succcurlyeq_F y$.
- Person i is complete dictator if he is complete dictator on X.

32 The Pareto Extension Rule (Sen 1970)

- A rule F is the Pareto extension rule on $Y \subset X$ if and only if $\forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in Y$,
  $x \succcurlyeq_F y \Longleftrightarrow \neg\,(y \succ_i x \;\forall i \in N)$.
- A rule F is the Pareto extension rule if and only if it is the Pareto extension rule on X.
- The Pareto extension rule satisfies BI and BP, but not CFR.

- Binary Neutrality holds on $Y \subset X$ if $\forall \succsim \in P(X)^n$ and $\forall x, y, z, w \in Y$, $\{i \in N : x \succsim_i y\} = \{i \in N : z \succsim_i w\}$ and $\{i \in N : x \precsim_i y\} = \{i \in N : z \precsim_i w\}$ imply $x \succsim_F y \Longleftrightarrow z \succsim_F w$.
- If Binary Neutrality holds on X, we say simply a rule F satisfies Binary Neutrality (BN).

34 Use of Non-Welfare Attributes (UNWA)

- If a rule uses non-welfare attributes, it must violate BN.
- Note: But not vice versa. The Borda rule violates BN but does not use any non-welfare attributes.

35 Use of Non-Welfare Attributes (UNWA)

- $X(x)$: the attribute set containing x.
- A rule F uses non-welfare attributes if either (i) or (ii) holds:
  (i) $\exists \succcurlyeq \in P(X)^n$ and $\exists\ x, y \in X$ s. t.
  $X(x) \neq X(y)$, $x \sim_i y \;\; \forall\ i$ and $x \nsim_F y$

Use of Non-Welfare Attributes (UNWA)

- (ii) $\exists \succsim \in P(X)^n$ and $\exists\, x, y, z, w \in X$ s. t.
  (ii-a) $\{i \in N : x \succsim_i y\} = \{i \in N : z \succsim_i w\}$
  and $\{i \in N : x \precsim_i y\} = \{i \in N : z \precsim_i w\}$;
  (ii-b) $z \notin X(x) \cup X(y)$ or $w \notin X(x) \cup X(y)$;
  and
  (ii-c) $x \succsim_F y \Longleftrightarrow z \succsim_F w$ does not hold.
- By (ii-b) $x = z$ & $y = w$ never happens.
- A rule F satisfies UNWA if it uses non-welfare attributes.

37 Theorem 1

(1) Suppose that there exists some X_λ with at least two elements. Then if a rule F satisfies CFR, BI and BP, there exists a person i who is decisive for any pair (x, y) except for all the pairs such that $\{x\} = X_\lambda$ and $\{y\} = X_{\lambda'}$.
(2) Suppose that any X_λ has at least two elements. Then if a rule F satisfies CFR, BI and BP, there exists dictator.

Example: (1) of Theorem 1

- $X = \{x, y, z, w\}$ where the attribute sets are $\{x, y\}, \{z\}$ and $\{w\}$.
- Person 1 is complete dictator on $\{x, y, z\}$ and $\{x, y, w\}$ and
- the Pareto extension rule governs on $\{z, w\}$.
- This rule satisfies CFR, BI and BP.
- 1 is not dictator but is decisive for all the pairs except for (z, w) and (w, z).

39 Example: (2) of Theorem 1

- Complete Dictatorial Rules.
- $\exists i \in N$ s.t. $x \succsim_F y \Longleftrightarrow x \succsim_i y$ $\forall \succsim \in P(X)^n, \forall x, y \in X$.
- This rule satisfies CFR, BI and BP.
- Person i is complete dictator.

40 Theorem 2

- If a rule satisfies CFR, BI, and UNWA, then it violates either FR or IP,
- and if there exist only two attribute sets, the rule satisfies FR and violates IP.

Rules satisfying CFR, BI, BP and UNWA.

attribute sets	(1) of Th. 1	(2) of Th. 1
only two ×IP and ○FR	Case 1	Case 2
three or more ×FR	Case 3	Case 4
three or more ×IP	Case 5	Case 6

- Case 5 is reduced to Case 3.

 Cases 1, 2 and 6:

- Each attribute set is indexed by X_τ $(\tau = 1, ..., t)$.
- For any $x \in X$, let $\tau(x) \in \{1, ..., t\}$ be such that $x \in X_{\tau(x)}$.

 Cases 1, 2 and 6:

- Given $\succcurlyeq \in P(X)^n$, $\forall x, y \in X$,
- $x \succ_L y \Longleftrightarrow$

$$\begin{cases} \exists k \in N \text{ s.t. } x \sim_i y \ \forall i \leq k-1 \ \& x \succ_k y \\ \text{or} \\ x \sim_i y \ \forall i \ \& \ \tau(x) > \tau(y). \end{cases}$$

- $x \sim_L y \Longleftrightarrow x \sim_i y \ \forall i \ \& \ \tau(x) = \tau(y)$.

Cases 1, 2 and 6:

- Let a rule F be such that $x \succcurlyeq_F y \Longleftrightarrow x \succcurlyeq_L y \;\; \forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in X$.
- F satisfies CFR, BI, BP and UNWA, and violates IP.

Case 3:

- $\forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in X$,
  if $x, y \in X_\lambda \cup X_{\lambda'}$ with $\#X_\lambda \geq 2$ or $\#X_{\lambda'}$
  ≥ 2,
  $x \succcurlyeq_F y \Longleftrightarrow x \succcurlyeq_1 y$
  otherwise $x \succcurlyeq_F y \Longleftrightarrow \neg (y \succ_i x \; \forall i \in N)$
- This rule satisfies CFR, BI, BP, UNWA and IP, and violates FR .

Case 4:

- Given at least three attribute sets.
- Let A, B, C be such that $A = \{x : \tau(x) = 1\}$, $B = \{x : \tau(x) = 2\}$, and $C = \{x : \tau(x) \geq 3\}$.

Case 4

- A binary relation $\geq_T$:

  $x >_T y \Longleftrightarrow$

  $[x \in A \& y \in B] \vee [x \in B \& y \in C] \vee$

  $[x \in C \& y \in A]$

  $x =_T y \Longleftrightarrow [x, y \in A] \vee [x, y \in B] \vee [x, y \in C]$

- $>_T$ has cycles such that $x >_T y >_T z >_T x$ where $x \in A$, $y \in B$ and $z \in C$.

Tightness: CFR: Non-welfare weighted Majority rule

- $N(x, y, \succsim) = \#\{i \in N : x \succ_i y\}$.
- $\forall \succsim \in P(X)^n$ and $\forall\ x, y \in X$,
- $x \succ_F y \iff N(x, y, \succsim) > N(y, x, \succsim)$ or $[N(x, y, \succsim) = N(y, x, \succsim)$ and $\tau(x) > \tau(y)]$
- $x \sim_F y \iff N(x, y, \succsim) = N(y, x, \succsim)$ and $\tau(x) = \tau(y)$.
-

CFR	BI	BP	UNWA
no	yes	yes	yes

Tightness: BI: Non-welfare weighted Borda rule

- $\beta(x,\succsim) = \sum_{i=1}^{n} \#\{y \in X : x \succsim_i y\}$.
- $k > 0$ s. t. $n + k > kt$. (to BP)
- $\forall \succsim \in P(X)^n$ and $\forall x, y \in X$,
- $x \succsim_F y \Longleftrightarrow$
  $\beta(x,\succsim) + k\tau(x) \geq \beta(y,\succsim) + k\tau(y)$
-

CFR	BI	BP	UNWA
yes	no	yes	yes

Tightness: BP: Non-welfare value first rule

- $\forall \succcurlyeq \in P(X)^n$ and $\forall x, y \in X$,
- $x \succ_F y \iff [\tau(x) > \tau(y)]$ or $[\tau(x) = \tau(y) \& x \succ_1 y]$
- $x \sim_F y \iff \tau(x) = \tau(y) \& x \sim_1 y$
CFR	BI	BP	UNWA
yes	yes	no	yes

Tightness: UNWA: Complete dictatorial rule

$$\exists i \in N s.t. \forall \succcurlyeq \in P(X)^n \text{ and } \forall x, y \in X$$

$$x \succcurlyeq_F y \Longleftrightarrow x \succcurlyeq_i y$$

CFR	BI	BP	UNWA
yes	yes	yes	no

52 Conclusion

- The study for non-welfarist social choice has not been studied since Sen (1979) that stressed its importance in the context of the criticism against Welfarism.
- To escape impossibility of dictatorship. Axiomatic study for Non-Welfare weighted Majority Rule and Borda rule should be recommended.