# A General Equilibrium Model with Firm Formation and Social Coalitional Equilibrium Structure

Ken Urai (Osaka University) · Kohei Shiozawa (Osaka University PD)

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## 1 Abstract

A General equilibrium model with the *firm formation* and the *determination of share holdings rates* is provided through a generalization of the *coalition production equilibrium* (CPE) framework in Böhm (1973) and the *social coalitional equilibrium* (SCE) concept in Ichiishi (1981).

## 2 Specific Features

1. The share holdings rates are treated as given (like prices) for all economic members although each *coalition* is allowed to change the rates as *SCE deviation strategy* so that the formation of firms and their share holdings rates are determined as a generalized sense of the social coalitional equilibrium.

2. Our equilibrium concept generalizes the notions of Böhm's CPE and Ichiishi's SCE in that (i) technologies of coalitions are assumed to depend on their investment levels, and (ii) *multiple coalition structures* having different investment purposes (such as different industries having independent coalition-deviation opportunities) are simultaneously determined.

3. Our model also gives an extension of the standard Arrow-Debreu private ownership economy (Debreu, 1959) and an answer to the *firm formation problem including the determination of share holdings rates*.

## 3 Agents, Coalitions and Technologies

There are $n$ agents, $\ell$ types of commodities, and $\lambda$ typese of investment purposes. Let $N = \{1, \ldots, n\}$ be the set of agents. For each investment purpose $t = 1, \ldots, \lambda$, a coalition structure $\mathfrak{T}^t$ (a partition of $N$) is determined as a generalized sense of the SCE equilibrium.

*Profit function* for $S \subset N$ under investment $(z_i^t)_{i\in S}$: $\pi^{t,S}(p, (z_i^t)_{i\in S}) \equiv \max\{p \cdot y | y \in Y^{t,S}(\sum_{i\in S} z_i^t)\}$.

*Income* for $i$ of $S$ under share-holdings rate $\theta^{t,S} \equiv (\theta_i^{t,S})_{i\in S}$: $I_i^{t,S}(p, (z_i^t)_{i\in S}; \theta^{t,S}) \equiv \theta_i^{t,S} \pi^{t,S}(p, (z_i^t)_{i\in S})$.

A list $(x_i, (z_i^t)_{t=1}^{\lambda})_{i=1}^n \in \prod_{i=1}^n (X_i \times (R_+^\ell)^\lambda)$ of agents' consumptions and investments is said to be *feasible* under coalition structure $\mathfrak{T} = (\mathfrak{T}^1, \ldots, \mathfrak{T}^\lambda)$, price $p$ and list of share holdings rates $\theta \equiv ((\theta^{t,S})_{S\in\mathfrak{T}^t})_{t=1}^{\lambda}$ if it satisfies the following two conditions.

**(Investment Feasibility)**: for each $i \in S$, $S \in \mathfrak{T}^t$ and $t = 1, \ldots, \lambda$, $\omega_i - \sum_{t=1}^{\lambda} z_i^t \in X_i$.

**(Budget Feasibility)**: for each $i \in N$, $p \cdot x_i \leqq p \cdot (\omega_i - \sum_{t=1}^{\lambda} z_i^t) + \sum_{t=1}^{\lambda} \sum_{S: i\in S\in\mathfrak{T}^t} I_i^{t,S}(p, (z_i^t)_{i\in S}; \theta^{t,S})$.

## 4 Equilibrium with Firm Formation

A list $((x_i, (z_i^t)_{t=1}^{\lambda})_{i=1}^n, \theta, p, \mathfrak{T})$ is an *equilibrium* state if the following three conditions are satisfied.

**(Feasibility Condition)** $(x_i, (z_i^t)_{t=1}^{\lambda})_{i=1}^n$ is feasible under $\mathfrak{T}$, $p$ and $\theta$.

**(Market-Clearing Condition)** $\sum_{i=1}^n x_i \in \sum_{i=1}^n \omega_i + \sum_{t=1}^{\lambda} \sum_{S\in\mathfrak{T}^t} \psi^{t,S}(p, (z_i^t)_{i\in S})$.

**(Market-Stability Condition)** There are no investment type $k \in \{1, \ldots, \lambda\}$, coalition $D \subset N$, list of consumptions $(\bar{x}_i)_{i\in D}$, list of investments $(\bar{z}_i^k)_{i\in D}$, and share-holdings rate $\bar{\theta}^{k,D}$ satisfying the following three conditions:

(i) $u_i(\bar{x}_i) > u_i(x_i)$ for all $i \in D$,

(ii) $\omega_i - \sum_{t\neq k} z_i^t - \bar{z}_i^k \in X_i$ for all $i \in D$,

(iii) $p \cdot \bar{x}_i \leqq p \cdot \omega_i + \sum_{t\neq k} \sum_{S: i\in S\in\mathfrak{T}^t} I_i^{t,S}(p, (z_i^t)_{i\in S}; \theta^{t,S}) + I_i^{k,D}(p, (\bar{z}_i^k)_{i\in D}; \bar{\theta}^{k,D})$ for all $i \in D$.

## 5 Theorems

**Lemma 1**: Let $((x_i, (z_i^t)_{t=1}^{\lambda})_{i=1}^n, \theta, p, \mathfrak{T})$ be an equilibrium state. Then, for any $i \in N$, $x_i$ maximizes $u_i$ subject to the constraint $\{x' \in X_i | p \cdot x' \leqq p \cdot (\omega_i - \sum_{t=1}^{\lambda} z_i^t) + \sum_{t=1}^{\lambda} \sum_{S: i\in S\in\mathfrak{T}^t} I_i^{t,S}(p, (z_i)_{i\in S}; \theta^{t,S})\}$.

The lemma says that if we redefine each agent's initial endowments as $\omega_i - \sum_{t=1}^{\lambda} z_i^t$ and recognize $Y^{t,S}(\sum_{i\in S} z_i^t)$ as the technology of each firm $S$, our equilibrium state is an equilibrium of the standard *Arrow-Debreu Private Ownership Economy.*

**Lemma 2**: Our equilibrium state can be identified with a *generalized Social Coalitional Equilibrium.*

For such an identification, we consider an additional agent, a *price and shareholdings rate manipulator.*

**Theorem**: A Generalized social coalitional equilibrium exists under a certain *balancedness condition.*

We investigate how the balancedness condition is satisfied or violated in our natural economic settings and assumptions.

## References

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