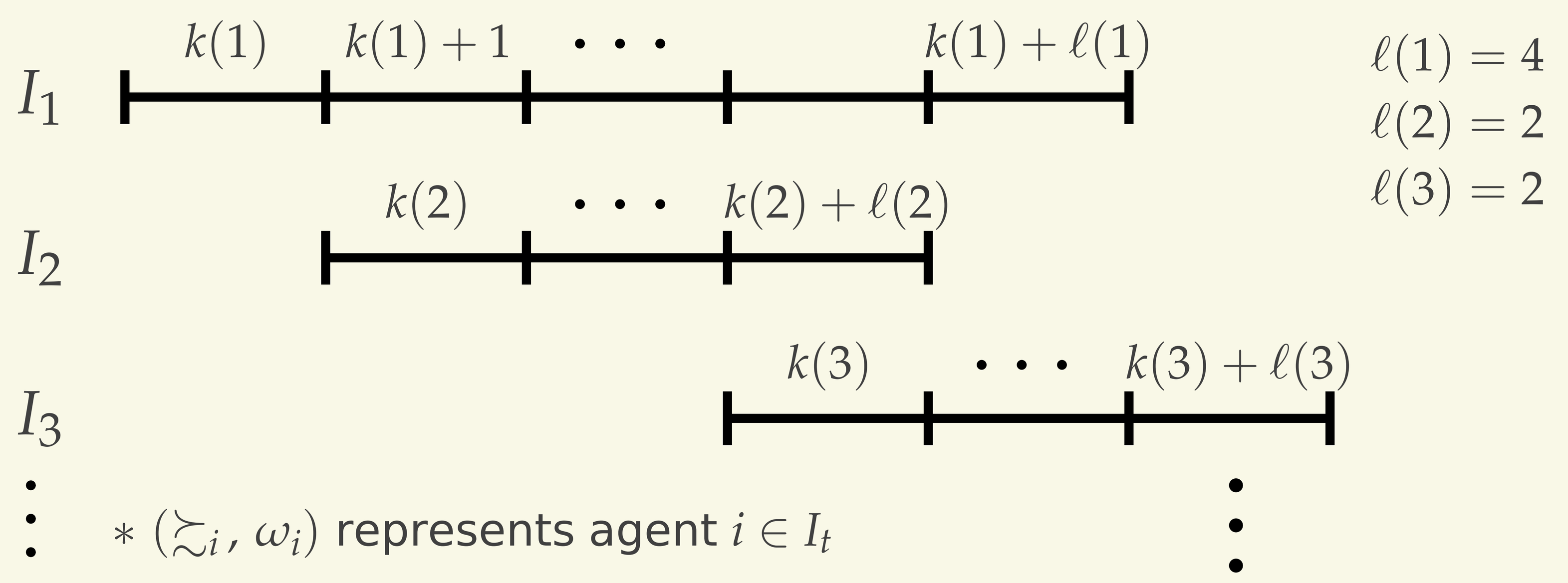
A Universal Implementability of Price-Money Mechanism for Overlapping-Generations Economies

Abstract

We axiomatically characterize **price-money mechanism** for overlapping-generations economies, using the core limit theorem of Urai and Murakami (2016). The category theoretic argument in Sonnenschein (1974) is generalized and reinterpreted as a **universal implementability** of price-money mechanism. Price-money mechanism is shown to be uniquely and efficiently implement all of other allocation mechanisms satisfying several basic axioms. We also show an isomorphism theorem, which says that we obtain a **minimum information space** having the universal implementability property.

Model



- An economy $\mathcal{E}$ is identified with the list, $(\{I_t\}, \{K_t\}, \{(\succsim_i, \omega^i)\})$, and $\mathcal{E}con$ denotes the set of all economies.
- $\mathcal{MW}alras(\mathcal{E})$ is the set of all **monetary equilibrium** allocations: the list of $p^* \in \mathcal{P}(\mathcal{E})$, feasible allocation $x^* = (x_i^* \in \boldsymbol{R}^{K_t})_{i \in \bigcup_{t \in N} I_t}$ and money supply function $M_{\mathcal{E}}^* : \boldsymbol{N} = \bigcup_{t=1}^{\infty} I_t \to \boldsymbol{R}_+$, where x_i^* is $\succsim_i$-greatest element in $\{x_i \in \boldsymbol{R}^{K_t} \mid p^* \cdot x_i \leqq p^* \cdot \omega_i + M_{\mathcal{E}}^*(i)\}$.
- Allocation x is **weakly Pareto-optimal** (**WPO**) if there is no y with $\sum_{t \in N} \sum_{i \in I_t} y_i = \sum_{t \in N} \sum_{i \in I_t} x_i$, such that $y_i = x_i$ except for finite agents, $y_i \succsim_i x_i$ for all i and $y_i \succ_i x_i$ for at least one i.
- **Finite core** of an economy $\mathcal{E}$, $\mathcal{F}core(\mathcal{E})$, is the set of feasible allocation that cannot be blocked by any finite coalition.

Message Mechanism

- A social choice correspondence is $g : \mathcal{E}con \ni \mathcal{E} \mapsto g(\mathcal{E}) \subset (\boldsymbol{R}_\infty)^N$.
- A **message mechanism** based on g is a triple (A, μ, f), where A is **message space**, $\mu : \mathcal{E} \mapsto \mu(\mathcal{E}) \subset A$ assigns **equilibrium messages**, $f : \mathcal{E}con \times A \to (\boldsymbol{R}_\infty)^N$ defines **responses** to message $a \in A$ for each agent in $\mathcal{E}$, satisfying $g(\mathcal{E}) = \{(f_i(\mathcal{E}, a))_{i=1}^{\infty} \mid a \in \mu(\mathcal{E})\}$.
- **Price-money mechanism** is defined by $(\mathcal{P} \times \mathcal{M}, \pi, e)$, where $\mathcal{P} \times \mathcal{M} = \{p \in \boldsymbol{R}^\infty \mid \exists [(x^t)_{t=1}^{\infty}] \in \Delta_{++},\ \mathrm{Pr}_{K(1)} p = x^1,\ \frac{\mathrm{Pr}_{K(t)} p}{\|\mathrm{Pr}_{K(t)} p\|} = x^t\} \times \{M \mid M : \mathcal{E}con \ni \mathcal{E} \mapsto M_{\mathcal{E}} \in R_+^N\}$, $\pi(\mathcal{E})$ is the set of all price-money equilibrium messages, e is the excess demand function such that $e_i(p, M_{\mathcal{E}})$ is the $\succsim_i$-greatest point in $\{x_i \in \boldsymbol{R}^{K_t} \mid p \cdot x_i \leqq p \cdot \omega_i + M_{\mathcal{E}}(i)\}$.

Replica Core Equivalence Theorem

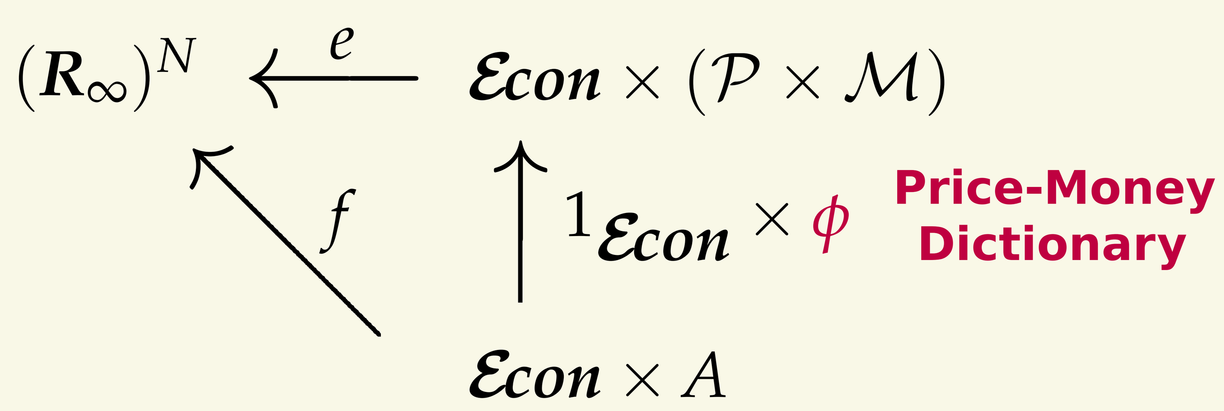
A feasible allocation x for $\mathcal{E}$ is a monetary competitive equilibrium allocation if and only if its $(m+n)$-fold replica allocation belongs to $\mathcal{F}core(\mathcal{E}^m(x) \oplus \mathcal{E}^n(\omega))$ for every $m \in \boldsymbol{N}$ and $n \in \boldsymbol{N}$. [Urai & Murakami 2016]

Axioms

- **Axiom I** (Idempotency)
  For each $\mathcal{E} \in \mathcal{E}con$ and $a \in A$, $f(\mathcal{E}(f(\mathcal{E}, a)), a) = f(\mathcal{E}, a)$.
- **Axiom S** (Sonnenschein)
  For each $(i_1, \mathcal{E}^1), \cdots, (i_m, \mathcal{E}^m)$, there exists an economy $\mathcal{E}_*$ including $\{i_1, \cdots, i_m\}$ such that a is an solution message for $\mathcal{E}_*$ for which the equilibrium list $(f^i(\mathcal{E}_*, a))_{i=1}^{\infty}$ is an extension of $(f^{i_s}(\mathcal{E}^s, a))_{s=1}^{m}$.

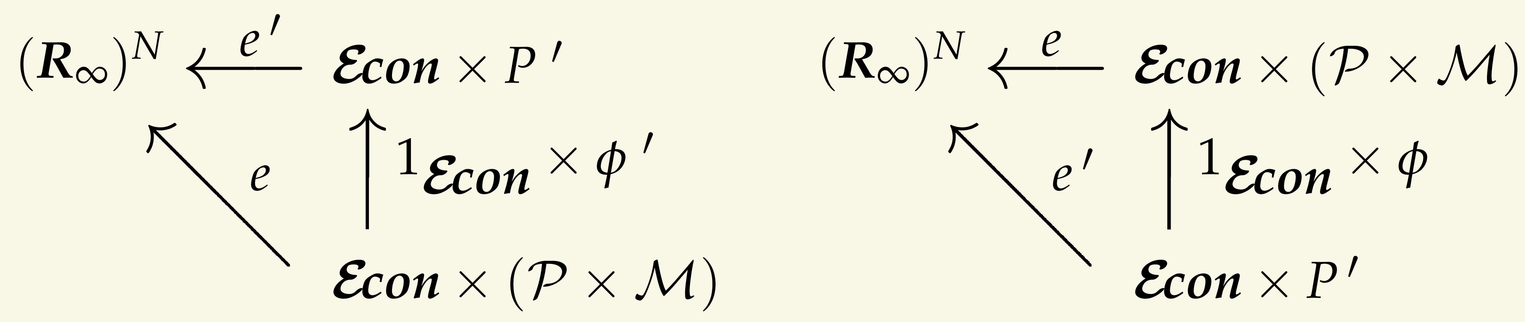
Universal Implementability Theorem

If (A, μ, f) is a message mechanism based on a social choice correspondence that is compatible with Fcore and WPO allocations, and if (A, μ, f) satisfies Axioms I and S, then there exists a unique function $\phi : A \to \mathcal{P} \times \mathcal{M}$ such that the next triangle commutes.



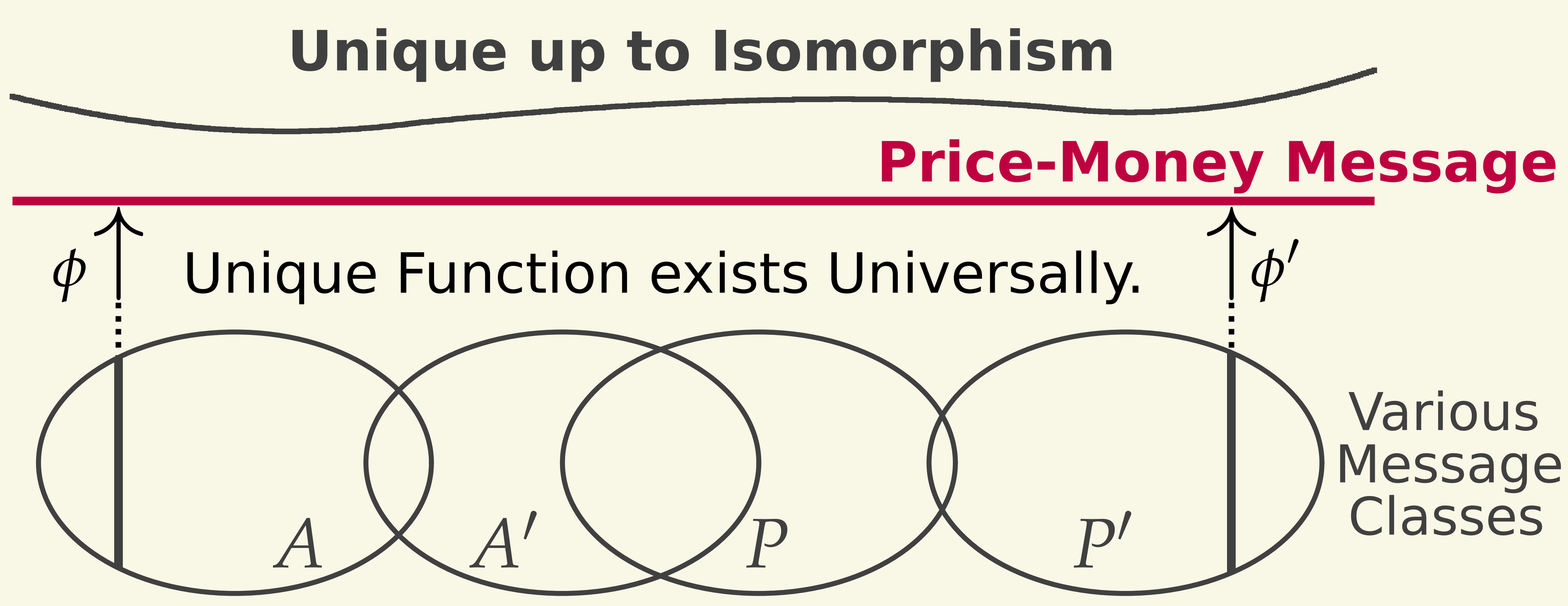
Isomorphism Theorem

Let $(\mathcal{P} \times \mathcal{M}, \pi, e)$ satisfy Axioms I and S. Then there exists isomorphism $h' : \mathcal{P} \times \mathcal{M} \to P'$ and $e = e' \circ [1_{\mathcal{E}con} \times h']$ for any Fcore-WPO compatible (P', π', e') satisfying Axioms I and S.



If the problem is restricted on spaces with topological (differentiable) structures and continuous (differentiable) mappings, then h' can be taken as a homeomorphism (diffeomorphism).

Conclusion



- **The universality and the efficiency property of price-money message mechanism is proved.**

References

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