Monetary Exchange and the Price Level in von Neumann's Multi-Sector Growth Framework

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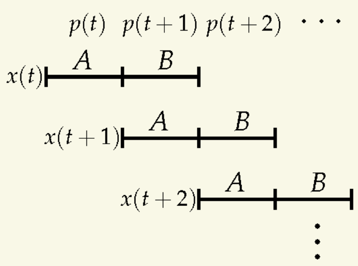
1 Background and Motivation

- We introduce money and price level into the von Neumann input-output model.
- Monetary exchange is treated as an activity or a process in the input-output framework (cf. Morishima 1977).
- Hence the money is intrinsically necessary for such processes and its neutrality cannot be assured.
- Price level is used to restore the neutrality of money.
- Money and credit (= bank-money) is treated as in overlapping generations model. In such case, the equilibrium state is known to be weakly Pareto optimal.
- To assure the existence of our monetary balanced growth path with constant expected deflation or inflation rate, we must generalize the lemma of Nikaido (see Morishima 1960) and use the Eilenberg-Montgomery fixed point theorem.

2 Model

A: input $\qquad\qquad$ B: output

$$\begin{array}{c} \\ \text{Bank} \\ \text{Buy} \\ \text{Sale} \\ \vdots \\ \text{Make} \end{array}
\begin{array}{c} \text{Money}\ \ 1\ \cdots\ n \\ \begin{bmatrix} a_{00} & 0 & \cdots & 0 \\ \frac{p_1}{p'_0(1+\iota)} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{m1} & \cdots & 0 \end{bmatrix} \end{array}
\Rightarrow
\begin{array}{c} \text{Money}\ \ 1\ \cdots\ n \\ \begin{bmatrix} b_{00} & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \frac{p_n(1+\iota)}{p_0} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{mn} \end{bmatrix} \end{array}$$



$I = \{i \mid 0, 1, \cdots, m\}$: index set of production processes

- 0-th process represents central bank (= government).

$J = \{j \mid 0, 1, \cdots, n\}$: index set of goods

- 0-th good denotes the bank-money.

$A = [A^0, \bar{A}\ ]$, $\ B = [B^0, \bar{B}\ ]$ $\quad\cdots$non negative matrices on $I \times J$

- A is input matrix and its element a_{ij} denotes the quantity of good j used in process i. B represents output matrix.

$x(t) = (x_0(t), \cdots, x_m(t)) \in R_+^{m+1}$: intensity of production in period t
$p(t) = (p_0(t), \cdots, p_n(t)) \in R_+^{n+1}$: price level in period t
$\alpha(t) = (1+$ growth rate from period t to $t+1)$
$\beta(t) = (1 +$ real interest rate$) = (1 + \iota(t))\tau(t)$
$\quad = (1 +$ nominal interest rate$) \times ($deflation factor$)$

- $\tau(t)$ is $1 +$ deflation rate, and $\tau(t) = \frac{p_0(t+1)}{p_0(t)}$ when $p_0(t) > 0$.

3 Equilibrium Conditions

An equilibrium is a sequence of variables satisfying the following formulas where $t = 0, 1, \cdots$ and $p_0(t) \neq 0$ for all t:

- $\alpha(t)x(t) \cdot A^0 - \tau(t)x(t) \cdot B^0 \leqq 0$,
- $\alpha(t)x(t) \cdot \bar{A}\ - x(t) \cdot \bar{B}\ \leqq 0$,
- $(1 + \iota(t))\tau(t)A \cdot p(t) - B \cdot p(t+1) \geqq 0$,
- $\alpha(t)(x(t) \cdot A^0)p_0(t+1) - \tau(t)(x(t) \cdot B^0)p_0(t+1) = 0$,
- $\alpha(t)(x(t) \cdot \bar{A}\ ) \cdot \bar{p}(t+1) - (x(t) \cdot \bar{B}\ ) \cdot \bar{p}(t+1) = 0$,
- $(1 + \iota(t))\tau(t)x(t)A \cdot p(t) - x(t)B \cdot p(t+1) = 0$.

4 Economic Assumptions

Generalized conditions by Kemeny et al.(1956).

- (i) Every row of A has at least one positive entry.
- (ii) Every column of B has at least one positive entry.
- $x(t) \cdot B \cdot p(t) > 0$.

Conditions for government process ensuring $p_0 > 0$.

- (iii) $A_0 = (\max\{\sum_{i=1}^m x_i(t+1)b_{i0} - (1 + \iota(t)) \sum_{i=1}^m \tau(t)x_i(t)a_{i0}, 0\}, \bar{A}_{\ 0})$,
- (iv) $B_0 = (\max\{(1 + \iota(t)) \sum_{i=1}^m \tau(t)x_i(t)a_{i0} - \sum_{i=1}^m x_i(t+1)b_{i0}, 0\}, \bar{B}_{\ 0})$,
- (v) $\bar{A}_{\ 0} \neq 0$,
- (vi) $\exists\ k \in J$, $b_{0k} > 0$, $a_{ik} > 0$ and $b_{ik} = 0$ for each $i = 1, \cdots, m$.

5 Balanced Growth Equilibrium

We will have $\alpha(t) = \alpha$, $\beta(t) = \beta$, $\iota(t) = \iota$, $\tau(t) = \tau$, $x(t) = x$ for all t. Instead of $p(t)$, consider the vectors $y(k) = (\frac{p_0(k)}{\tau^{k-t}}, p_1(k), \cdots, p_m(k))$, $y(t) = p(t)$ and $y(t+1) = (\frac{p_0(t+1)}{\tau(t)}, p_1(t+1), \cdots, p_m(t+1)) = (p_0(t), p_1(t+1), \cdots, p_m(t+1))$, then we will have $y(t) = y(t+1) = y$.

(1) $\alpha x \cdot A - x \cdot BI(\tau) \leqq 0$,
(2) $(1 + \iota)\tau A \cdot y - BI(\tau) \cdot y \geqq 0$,
(3) $\alpha(x \cdot A) \cdot y - (x \cdot BI(\tau)) \cdot y = 0$,
(4) $(1 + \iota)\tau x A \cdot y - xBI(\tau) \cdot y = 0$,
(5) $x \cdot BI(\tau) \cdot y > 0$.

$$I(\tau) = \begin{bmatrix} \tau & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$BI(\tau) = [\tau B_0, \bar{B}\ ]$$

The nominal interest rate should be $\iota \geqq 0$. Even in such case, $\tau \leqq 1$ would be compatible with $\beta < 1$.

6 Lemma

A mapping $(x^*, y^*, \lambda^*) \mapsto S \times T(x^*, y^*, \lambda^*)$ is upper semicontinuous on $X \times Y \times [0, 1]$ into itself, where $S \times T(x^*, y^*, \lambda^*) = \{x \mid x$ satisfies (6) and (8) under $(x^*, y^*, \lambda^*)\} \times \{(y, \lambda) \mid (y, \lambda)$ satisfies (7) and (8) under $(x^*, y^*, \lambda^*)\}$. Furthermore, $S \times T$ is contractible valued.

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From (3) - (5), $\alpha = \beta = (1 + \iota)\tau > 0$. Let us define $\lambda = \alpha/(1 + \alpha)$.

- (6) $\lambda x \cdot A(x^*, y^*, \lambda^*) - (1 - \lambda)x \cdot B(x^*, y^*, \lambda^*)I(\lambda^*) \leqq 0$,
- (7) $\lambda A(x^*, y^*, \lambda^*) \cdot y - (1 - \lambda)B(x^*, y^*, \lambda^*)I(\lambda^*) \cdot y \geqq 0$,
- (8) $v[\lambda A(x^*, y^*, \lambda^*) - (1 - \lambda)B(x^*, y^*, \lambda^*)I(\lambda^*)] = 0$.

7 Existence Theorem

THEOREM: Under assumptions (i) - (vi), a solution exists for inequalities (1) - (5) such that $y_0 > 0$.

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From the Eilenberg-Montgomery fixed point theorem, mapping $(x^*, y^*, \lambda^*) \mapsto S \times T(x^*, y^*, \lambda^*)$ of Lemma has a fixed point.

References

[1] Kemeny, Morgenstern and Thompson (1956): "A Generalization of the von Neumann model of an Expanding Economy," Econometrica. [2] Morishima (1960): "Economic Expansion and the Interest Rate in Generalized von Neumann Models," Econometrica. [3] Morishima (1977): "Walras' Economics: A Pure Theory of Capital and Money," Cambridge Univ. Press. [4] Nikaido (1968): "Convex Structures and Economic Theory," Academic Press. [5] von Neumann (1937): "A Model of General Economic Equilibrium," RES.