Existence and Core Equivalence Theorems for Competitive Equilibria of an Exchange Economy with Infinite Time Horizon

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The existence of competitive equilibrium for a large exchange economy over the commodity space ℓ^∞ will be discussed. Let $\beta > 0$ be a given positive number. We will assume that the consumption set X of each consumer is the set of nonnegative vectors whose coordinates are bounded by β,

$$X = \{\boldsymbol{x} = (x^t) \in \ell^\infty |\ 0 \le x^t \le \beta \text{ for } t \ge 1\}.$$

As usual, a preference $\succsim$ is a complete, transitive and reflexive binary relation on X. We denote $(\xi, \zeta) \in \succsim$ by $\xi \succsim \zeta$. $\xi \prec \zeta$ means that $(\xi, \zeta) \notin \succsim$. Let $\mathcal{P} \subset 2^{X \times X}$ be the collection of allowed preference relations. Let $(A, \mathcal{A}, \nu)$ be a probability space of the consumers which is assumed to be complete and atomless. For each $a \in A$, we have a map which assigns a his/her preference $\succsim_a \in \mathcal{P}$. We assume

Assumption (PR).

(*i*) $\succsim \in \mathcal{P}$ is complete, transitive and reflexive,

(*ii*) (monotonicity) for each $\xi \in X$ and $\zeta \in X$ such that $\xi < \zeta$, $\xi \prec \zeta$,

(*iii*) (Measurability) The preference assignment map is measurable in the sense that $\{(a, \xi, \zeta) \in A \times X \times X |\ \xi \succsim_a \zeta\} \in \mathcal{A} \times \mathcal{B}(X) \times \mathcal{B}(X)$.

For the existence of competitive equilibrium, we need

Assumption (CV) (Convexity). For $\succsim \in \mathcal{P}$, the set $\{\xi \in X |\ \xi \succsim \zeta\}$ is convex for all $\zeta \in X$.

We denote the set of all endowmet vectors by Ω and assume that it is of the form

$$\Omega = \{\omega = (\omega^t) \in \ell^\infty |\ 0 \le \omega^t \le \gamma \text{ for } t \ge 1\},$$

for some positive $\gamma (< \beta)$. The set Ω is also a compact metric space by the same reason as the space X.

An endowment assignment map ω is a Borel measurable map from A to Ω, $a \mapsto \omega(a) \in \Omega$. By Fact 8 and the assumption for Ω, the map ω is Gel'fand integrable. The following assumption which means that every commodity is available in the market is standard.

Assumption (PE) (Positive total endowment). $\int_A \omega(a) d\nu \ggg \mathbf{0}$.

An economy $\mathcal{E}$ is a mapping $\mathcal{E} : A \to \mathcal{P} \times \Omega$ defined by $a \mapsto (\succsim_a, \omega(a))$. An allocation is a Gel'fand integrable map $\xi : A \to X$. An allocation is feasible if $\int_A \xi(a)d\nu \leq \int_A \omega(a)d\nu$. An economy $\mathcal{E}$ is called *irreducible* if it satisfies the next condition.

Assumption (IR) (Irreducibility). For every measurable partion $\{A_1, A_2\}$ of A and for every feasible alocation ξ, there exists an allocation ϕ such that $\int_{A_2}(\omega(a) - \phi(a))d\nu + \int_{A_1} \xi(a)d\nu \in \int_{A_1}\{\zeta \in X|\ \xi(a) \prec_a \zeta\}d\nu$ whenever $\int_{A_1}\{\zeta \in X|\ \xi(a) \prec_a \zeta\}d\nu \neq \emptyset$.

Remark. The irreducible condition was introduced by McKenzie (1959). It was used in the continuum of consumers model by Yamazaki (1981). This condition was also used for models with infinite time horizon by Boyd and McKenzie (1993). See also McKenzie (2002).

The definition of the competitive equilibrium is standard.

Definition 1. A pair $(\boldsymbol{p}, \xi)$ of a price vector $\boldsymbol{p} \in \ell^1_+$ with $\boldsymbol{p} \neq \mathbf{0}$ and an allocation $\xi : A \to X$ is called a competitive equilibrium of the economy $\mathcal{E}$ if the following conditions hold,

(E-1) $\boldsymbol{p}\xi(a) \leq \boldsymbol{p}\omega(a)$ and $\xi(a) \succsim_a \zeta$ whenever $\boldsymbol{p}\zeta \leq \boldsymbol{p}\omega(a)$ a.e,

(E-2) $\int_A \xi(a)d\nu = \int_A \omega(a)d\nu$.

The main result of this paper now reads

Theorem 1. Let $\mathcal{E}$ be an economy which satisfies the assumptions (PR), (CV), (PE) and (IR). Then there exists a competitive equilibrium $(\boldsymbol{p}, \xi)$ for $\mathcal{E}$.

We now define the core. Let $\mathcal{E}$ be an economy and ξ an allocation.

Definition 2. A measurable set $C \subset A$ is said to block the allocation ξ if there exists a measurable map $\zeta : A \to X$ such that

(C-1) $\int_C \zeta(a)d\nu \leq \int_C \omega(a)d\nu$,

(C-2) $\xi(a) \prec_a \zeta(a)$ on C.

A feasible allocation ξ belongs to a core of an economy $\mathcal{E}$ if there exist no measurable sets $C \subset A$ with $\nu(C) > 0$ which blocks ξ.

The second main result of this paper now reads

Theorem 2. Let $\mathcal{E}$ be an economy which satisfies the assumptions (PR), (PE) and (IR). A feasible allocation ξ belongs to the core of the economy $\mathcal{E}$ if and only if there exists a price vector $\boldsymbol{p} \in \ell^1_+$ with $\boldsymbol{p} \neq \mathbf{0}$ such that $(\boldsymbol{p}, \xi)$ is a competitive equilibrium for $\mathcal{E}$.