HETEROGENEITY AND AGGREGATION

. MACROECONOMIC INDETERMINACY IF DISTRIBUTIONS OF INDIVIDUAL CHARACTERISTICS IS ARBITRARY

EX: IN EXCHANGE ECONOMY, AGGREGATE EXCESS DEMAND IS ARBITRARY ON ANY COMPACT SET OF POSITIVE PRICES IF DISTRIBUTION OF PREFERENCES AND ENDOWMENTS IS ARBITRARY (SONNENSCHEIN 1973, 1974; DEBREU, 1974,...)

TRUE EVEN IF PREFERENCES ARE HOMOTHETIC (MANTEL, McFADDEN, 1974,...)

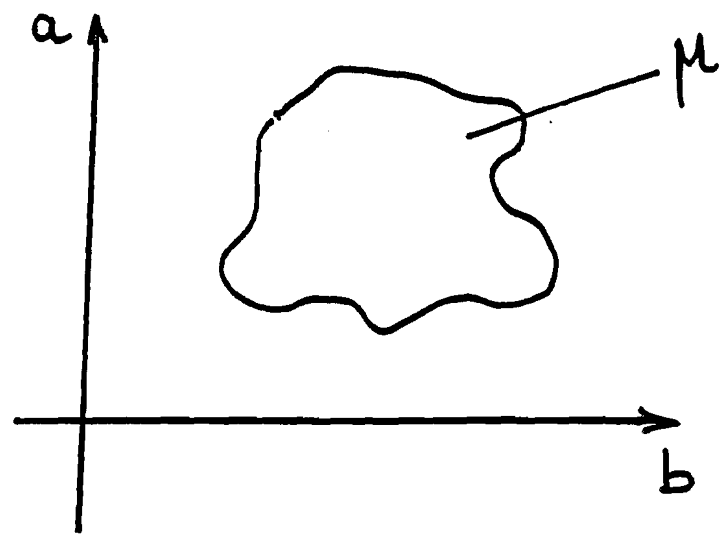
- **<u>PROGRAM:</u>** SPECIFIC MACROECONOMIC PROPERTIES RESULTING FROM PARTICULAR DISTRIBUTIONS OF INDIVIDUAL CHARACTERISTICS?

- EMPHASIS ON <u>SHAPES</u> OF DISTRIBUTIONS NOT ON RESTRICTIONS ON SUPPORT (HETEROGENEITY)

→ MONOTONICITY OF AGGREGATE DEMAND? ("LAW OF DEMAND", HILDENBRAND, JERISON, KNIEP, ...)

→ DISPERSED DISTRIBUTIONS OF PREFERENCES (GRANDMONT, QUAH, KNIEP, ...)

→ MONOTONE AGGREGATE DEMAND



- AGGREGATE DEMAND (PRICE INDEPENDENT INCOMES)

$$F(p) = \int_{A \times \mathbb{R}_+} f(a, p, b) \, d\mu$$

- MONOTONICITY ("LAW OF DEMAND")?

$$(q-p) \cdot (F(q) - F(p)) < 0 \quad \forall p \neq q$$

- SUFFICIENT CONDITION: JACOBIAN MATRIX $\partial_p F(p)$ IS NEGATIVE SEMI-DEFINITE ($v' \cdot \partial_p F(p) v < 0 \quad \forall v \in \mathbb{R}^n \neq 0$)

(PROOF: $(q-p) \cdot (F(q) - F(p)) = (q-p) \cdot (\partial_p F)(q-p) < 0$ WHERE $\partial_p F$ EVALUATED AT SOME POINT IN SEGMENT $[p, q]$ (TAYLOR). Q.E.D.)

- AGGREGATE DEMAND

$$F(p) = \int_{A \times R_+} f(a,p,b)\, d\mu$$

- SLUTSKY DECOMPOSITION

$$\partial_p f(a,p,b) = Sf(a,p,b) - Mf(a,p,b)$$

$$\partial_p F(p) = S(p) - M(p)$$

WHERE
$$S(p) = \int Sf(a,p,b)\, d\mu$$
$$M(p) = \int Mf(a,p,b)\, d\mu$$

- MONOTONICITY OF AGGREGATE DEMAND, $\partial_p F(p)$ NEGATIVE SEMI-DEFINITE, OBTAINS IF

H.1. AVERAGE SLUTSKY MATRIX $S(p)$ IS NEGATIVE DEFINITE (WILL OBTAIN IF INDIVIDUAL PREFERENCE MAXIMIZATION, OR WEAK AXIOM, $v' \cdot Sf(a,p,b) v < 0 \quad \forall v \neq 0,\ v \perp p$)

H.2. AVERAGE INCOME EFFECTS MATRIX $M(p)$ IS POSITIVE SEMI-DEFINITE ($v' \cdot M(p) v > 0 \quad \forall v \in \mathbb{R}^n,\ v \neq 0$)

- MACRO ECONOMIC LAW OF DEMAND

$\Rightarrow$ UNIQUENESS AND TATÔNNEMENT STABILITY OF COMPETITIVE EQUILIBRIUM IN EXCHANGES ECONOMIES WITH COLINEAR ENDOWMENTS (PRICE NORMALIZATION, $p\cdot\bar{\omega}=1$ $\Rightarrow$ PRICE INDEPENDENT INCOMES)

EXAMPLE:

EXCHANGE ECONOMY, HOMOTHETIC PREFERENCES DISTRIBUTIONS OF PREFERENCES AND OF ENDOWMENTS ARE INDEPENDENT ($\Leftrightarrow$ EACH AGENT HAS AVERAGE ENDOWMENT $\bar{\omega}$ $\Rightarrow$ PRICE INDEPENDENT INCOMES THROUGH $p\cdot\bar{\omega}=1$)

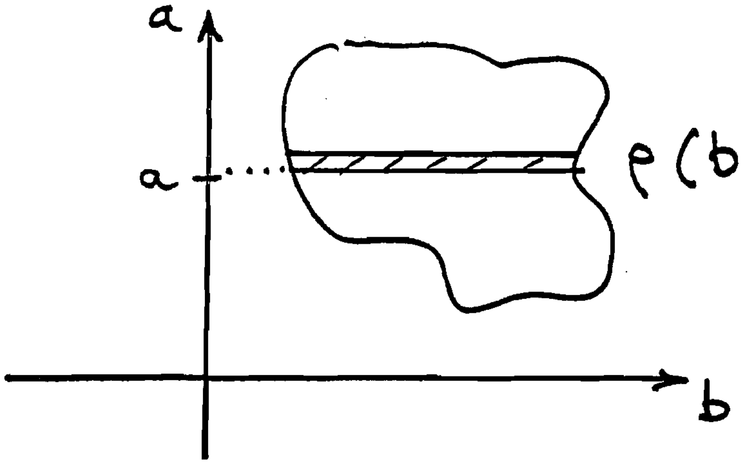
$\Rightarrow$ $M(p)$ POSITIVE SEMIDEFINITE

$$Mf(a,p,b)=\frac{\partial f}{\partial b}(a,p,b)\,[f(a,p,b)]'=f\,[f]'/b$$

$$v'\cdot M(p)\,v=\left[\int \frac{1}{b}(v'\cdot f)(f\cdot v)\,d\mu\right]$$

$$=\left[\int \frac{1}{b}(f\cdot v)^2\,d\mu\right] \quad > 0$$

- HILDENBRAND (1983): FOR GIVEN a,



- $\overline{M}f(a,p) = \int_{\mathbb{R}_+} Mf(a,p,b)\,\rho(b|a)\,db$

IS POSITIVE SEMI-DEFINITE IF CONDITIONAL INCOME DISTRIBUTION HAS C^1 DECREASING DENSITY $\rho'(b|a) < 0$.

$\Rightarrow M(p)$ POSITIVE SEMI-DEFINITE BY INTEGRATION OVER a (ADDITIVE PROPERTY)

SINCE $Mf(a,p,b) = \frac{\partial f}{\partial b}(a,p,b)\,[f(a,p,b)]'$

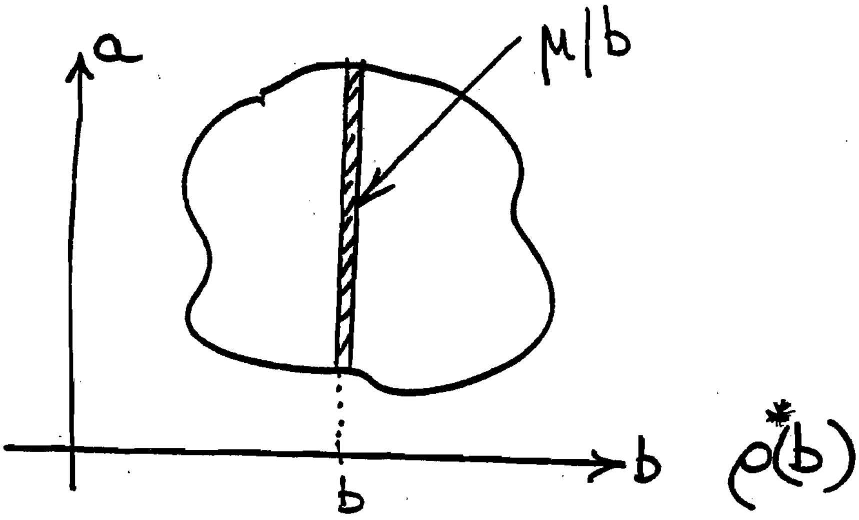
$$v' \cdot \overline{M}f(a,p) v = \int_{\mathbb{R}_+} \left(v' \cdot \frac{\partial f}{\partial b}\right)(f \cdot v)\,\rho(b|a)\,db$$

$$= \frac{1}{2}\int_{\mathbb{R}_+} \frac{\partial}{\partial b}\left[(f\cdot v)^2\right]\rho(b|a)\,db$$

(INTEG. BY PARTS) $= -\frac{1}{2}\int_{\mathbb{R}_+} (f \cdot v)^2\,\rho'(b|a)\,db > 0.$

. WEAKER CONDITIONS?

→ HILDENBRAND (1994): AVERAGE MATRIX OF INCOME EFFECTS M(p) IS POSITIVE DEFINITE (H.2) IF

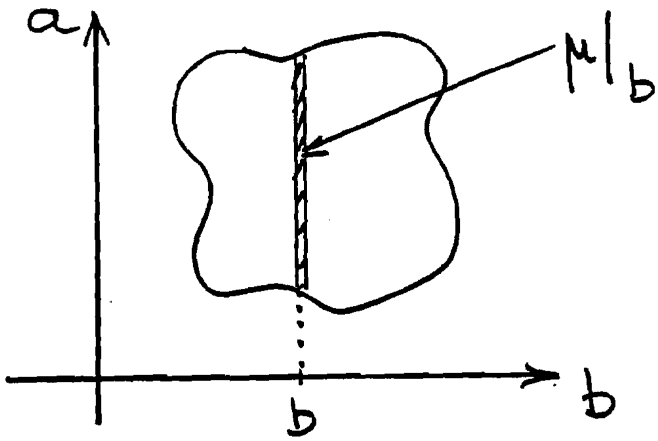


<u>H.2.a.</u> AVERAGE DISPERSION OF DEMANDS CONDITIONAL ON INCOME LEVEL b IS INCREASING (JERISON, 1982, 1984)

<u>H.2 b.</u> <u>MARGINAL</u> INCOME DISTRIBUTION HAS DECREASING DENSITY $\rho^*(b)$

→ NON DECREASING DENSITIES OF INCOME DISTRIBUTION POSSIBLE IF STRONG SUBSTITUTION EFFECTS (S(p) "STRONGLY" NEGATIVE SEMI-DEFINITE): HILDENBRAND; MARHUENDA, 1995; ...

→ DISPERSED DISTRIBUTIONS OF TASTES



- MACROECONOMIC PROPERTIES WITH SPECIFIC DISTRIBUTIONS OF "TASTES" CONDITIONAL ON INCOME, $\mu|_b$?

- BECKER (1962): "IRRATIONAL" UNIFORM DISPERSION OF EXPENDITURES ⇒ COBB-DOUGLAS WITH EQUAL WEIGHTS

- IMPORTANCE OF COMPATIBILITY WITH "RATIONALITY" (LINEAR STRUCTURES ON "TASTES")

→ AFFINE TRANSFORMATIONS

- INDIVIDUAL EXPENDITURES $w_i(p, b_i)$ ("TYPE i")
- EQUIVALENCE CLASS OF "TASTES" INDEXED BY $\alpha = (\alpha_1, \dots, \alpha_\ell) \in \mathbb{R}^\ell$, WHERE EXPENDITURES AT (p, b_i) ARE "AS IF" PRICES WERE $e^\alpha \otimes p$:

$$w_i(\alpha, p, b_i) = w_i(e^\alpha \otimes p, b_i), \quad e^\alpha \otimes p = (e^{\alpha_1} p_1, \dots, e^{\alpha_\ell} p_\ell)$$

- AGGREGATE EXPENDITURES (COND. ON "TYPE i")

$$W_i(p) = \int_{\mathbb{R}^\ell} w_i(\alpha, p, b_i)\, g_i(\alpha)\, d\alpha$$

- COMPATIBLE WITH "RATIONALITY": $u(x) \to u(e^{-\alpha} \otimes x)$

MAX $u(x)$	MAX $u(e^{-\alpha} \otimes x)$
$p \cdot x = b$	$p \cdot x = b$
	$(e^\alpha \otimes p) \cdot (e^{-\alpha} \otimes x)$
$\Rightarrow f(p, b)$	$\Rightarrow f(\alpha, p, b) = e^\alpha \otimes f(e^\alpha \otimes p, b)$

- "DISPERSION OF TASTES" WITHIN AFFINE EQUIVALENCE CLASS ("FLAT" DENSITY $g_i(\alpha)$) STABILIZES AGGREGATE EXPENDITURES (GRANDMONT, 1992)

$$\partial_{\text{Log}\, p_k} W_{ik}(p) = \int_{\mathbb{R}^\ell} \partial_{\alpha_k}(w_{ik}(\alpha, p, b_i))\, g_i(\alpha)\, d\alpha$$

$$= - \int_{\mathbb{R}^\ell} w_{ik}(\alpha, p, b_i)\, \partial_{\alpha_k}(g_i(\alpha))\, d\alpha$$

$$| \qquad | \leqq b_i \int_{\mathbb{R}^\ell} |\partial_{\alpha_k} g_i(\alpha)|\, d\alpha \leqq b_i\, m_{ik}$$

- DEMAND FUNCTION INVARIANT BY AFFINE α-TRANSFORMATIONS, $w(p,b) \equiv w(e^\alpha \otimes p, b)\ \forall \alpha$, IFF COBB-DOUGLAS

- "FLAT" DENSITY $g_i(\alpha)$ (m_{ik} SMALL) MEANS CLOSE TO LEBESGUE MEASURE ON $\mathbb{R}^\ell$, I.E. TO UNIQUE INARIANT HAAR MEASURE ON GROUP OF α-TRANSFORMATIONS

⇒ "FLAT" DENSITY $g_i(\alpha)$ (m_{ik} SMALL) GENERATES CONDITIONAL MARKET DEMAND THAT SATISFIES LAW OF DEMAND, DIAGONAL DOMINANCE, ...

⇒ TRUE ALSO FOR MARKET DEMAND GIVEN BY AGGREGATION OVER "TYPES i" (ADDITIVE PROPERTIES)

$$W(p) = \int_I W_i(p)\, \mu(di)$$

⇒ IN EXCHANGE ECONOMIES WHERE INCOME IS PRICE DEPENDENT, $b_i = p \cdot \omega_i$: GROSS SUBSTITUTABILITY OF AGGREGATE EXCESS DEMAND, UNICITY AND STABILITY OF COMPETITIVE EQUILIBRIUM, WEAK AXIOM BETWEEN EQUILIBRIUM PRICE p^* AND ANY OTHER p NOT COLINEAR TO p^*

→ **APPLICATION** : DISPERSION OF BELIEFS IN COMPLETE ASSET MARKETS (CALVET, LEMAIRE, GRANDMONT, 1999)

- EXCHANGE ECONOMY ON ARROW-DEBREU SECURITIES $s=1,\dots,S$. PRICE SYSTEM $q \in \mathbb{R}^S_{++}$

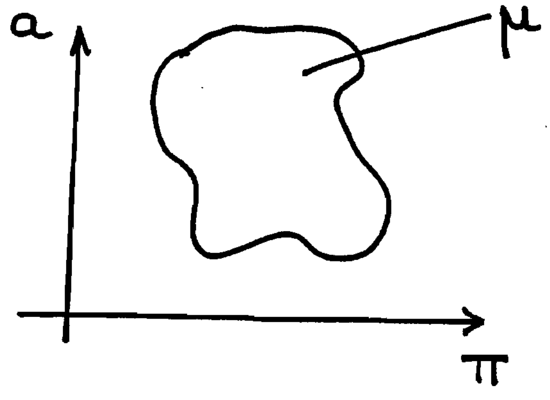
- AGENT (a,π) CHOOSES PORTFOLIO $y_a=(y_{a1},\dots,y_{aS}) \geq 0$ OF A-D SECURITIES SO AS TO MAXIMIZE

$$E_\pi\left[u_{as}(y_{as})\right]=\sum_s \pi_s u_{as}(y_{as}) \text{ S.T. } q\cdot y_a=b_a$$

$\Rightarrow y_s(a,q,b_a,\pi)$ OR INDIVIDUAL EXPENDITURES

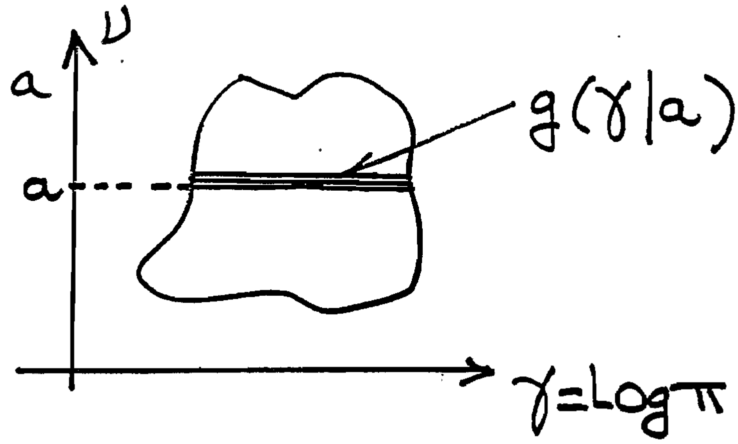
$$w_s(a,q,b_a,\pi)=q_s y_s(a,q,b_a,\pi)$$

- AGGREGATE EXPENDITURES



$$W(q)=\int_{A\times\Pi} w(a,q,b_a,\pi)\,d\mu$$

- CONDITIONAL AGGREGATE EXPENDITURES



$$W_a(q) = \int_\Gamma w(a, q, b_a, \gamma)\, g(\gamma|a)\, d\gamma$$

$$W(q) = \int_A W_a(q)\, d\nu$$

- "DISPERSION OF BELIEFS" ("FLAT" DENSITIES $g(\gamma|a)$) MAKES CONDITIONAL AGGREGATE EXPENDITURES LESS SENSITIVE TO ASSET PRICES, I.E. $\left|\frac{\partial W_{ar}}{\partial \operatorname{Log} q_s}(q)\right|$ SMALL.

- TRUE ALSO FOR AGGREGATE EXPENDITURES

$$\left(\left|\frac{\partial W_r}{\partial \operatorname{Log} q_s}(q)\right| \leq \int_A \left|\frac{\partial W_{ar}}{\partial \operatorname{Log} q_s}(q)\right| d\nu \right)$$

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1) $$\frac{\partial w_r}{\partial \operatorname{Log} q_s}(a,q,b_a,\gamma) = -(1-\rho_{as})\frac{\partial w_r}{\partial \gamma_s}(a,q,b_a,\gamma)$$

WITH $\rho_{as} = -y_{as}u''_{as}(y_{as})/u'_{as}(y_{as})$ = RELATIVE DEGREE OF RISK AVERSION

(PROOF: BY DIRECT DIFFENCIATION OF FOC

$$\frac{\pi_s}{q_s}u'_{as}(y_{as}) = \frac{\pi_r}{q_r}u'_{ar}(y_{ar})\ )$$

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2) $\forall\, r \neq s$

$$\frac{\partial W_{ar}}{\partial \operatorname{Log} q_s}(q) = -\int_\Gamma (1-\rho_{as})\frac{\partial w_r}{\partial \gamma_s}(a,q,b_a,\gamma)\,g(\gamma|a)\,d\gamma$$

$$< 0$$

$$| \quad | \leqq -\eta_a \int_\Gamma \frac{\partial w_r}{\partial \gamma_s}(a,q,b_a,\gamma)\,g(\gamma|a)\,d\gamma$$

WITH $|1-\rho_{as}| \leqq \eta_a$

$$\leqq \eta_a \int_\Gamma w_r(a,q,b_a,\gamma)\frac{\partial g}{\partial \gamma_s}(\gamma|a)\,d\gamma$$

$$\leqq \eta_a b_a \int_\Gamma \left|\frac{\partial g}{\partial \gamma_s}(\gamma|a)\right| d\gamma \leqq \eta_a b_a m_{as}$$

$$\Rightarrow \left|\frac{\partial W_r}{\partial \log q_s}(q)\right| \leq \int_A \eta_a b_a m_{as} \, d\nu$$

$\Rightarrow$ "DISPERSION OF BELIEFS" ("FLAT" DENSITIES $g(\gamma | a)$, m_{as} SMALL $\forall a, s$) MAKES AGGREGATE EXPENDITURES ON A-D SECURITIES LESS SENSITIVE TO ASSET PRICES

$\Rightarrow$ IN EXCHANGE ECONOMIES WHERE INCOME $b_a = p \cdot \omega_a$ IS PRICE DEPENDENT: GROSS SUBSTITUTABILITY OF AGGREGATE EXCESS DEMAND, UNICITY AND STABILITY OF COMPETITIVE EQUILIBRIUM, WEAK AXIOM BETWEEN EQUILIBRIUM PRICESYSTEM q^* AND ANY OTHER q NOT COLINEAR TO q^*

→ HOMOTHETIC TRANSFORMATIONS

- INDIVIDUAL EXPENDITURES $w_i(p, b_i)$ ("TYPE i")

- EQUIVALENCE CLASS OF HETEROGENEOUS "TASTES" INDEXED BY $\beta \in \mathbb{R}$, WHERE EXPENDITURES AT (p, b_i) ARE "AS IF" PRICES WERE $e^{\beta} p$:

$$w_i(\beta, p, b_i) = e^{\beta} w_i(p, e^{-\beta} b_i)$$

- AGGREGATE EXPENDITURES (COND. ON "TYPE i"):

$$W_i(p, b_i) = \int_{\mathbb{R}} w_i(\beta, p, b_i)\, g_i(\beta)\, d\beta$$

- INTRODUCED BY JERISON (1982), GRANDMONT (1987, SPECIFIC CLASS OF DENSITIES $g_i(\beta)$ YELDING LAW OF DEMAND AS IN HILDENBRAND, 1983)

- QUAH (1997): "DISPERSION OF TASTES" WITHIN HOMOTHETIC EQUIVALENCE CLASS ("FLAT" DENSITY $g_i(\beta)$) MAKES AGGREGATE EXPENDITURES NEARLY LINEAR IN INCOME:

$$\frac{\partial W_i}{\partial \operatorname{Log} b_i}(p,b_i) - W_i(p,b_i) = -\int_{\mathbb{R}} \partial_\beta w_i(\beta,p,b_i)\, g_i(\beta)\, d\beta$$

$$= \int_{\mathbb{R}} w_i(\beta,p,b_i)\, g_i'(\beta)\, d\beta$$

$$|\qquad| \leq b_i \int_{\mathbb{R}} |g_i'(\beta)|\, d\beta \leq b_i\, m_i$$

- DEMAND FUNCTION $w_i(p,b_i)$ INVARIANT THROUGH HOMOTHETIC TRANSFORMATIONS $\forall \beta$ IFF LINEAR IN INCOME
- DENSITY $g_i(\beta)$ "FLAT" (m_i SMALL) IFF CLOSE TO LEBESGUE MEASURE ON $\mathbb{R}$, I.E. TO UNIQUE INVARIANT HAAR MEASURE ON GROUP OF HOMOTHETIC β-TRANSFORMATIONS

• IMPLICATIONS (QUAH, 1997):

MARKET DEMAND DEFINED BY AGGREGATE EXPENDITURES

$$W(p) = \int_I W_i(p, b_i)\, \mu(di)$$

1. IF WEAK AXIOM SATISFIED (INDIVIDUAL SLUTSKY MATRICES ARE NEGATIVE SEMI-DEFINITE), "FLAT" DENSITIES $g_i(\beta)$ (m_i SMALL) IMPLY LAW OF DEMAND (JACOBIAN MATRIX $\partial_p W(p)$ NEGATIVE SEMIDEFINITE)
2. IN EXCHANGE ECONOMIES WITH $b_i = p \cdot \omega_i$, IF IN ADDITION, DISTRIBUTIONS OF PREFERENCES AND OF ENDOWMENTS ARE INDEPENDENT, MARKET DEMAND SATISFIES LAW OF DEMAND FOR NORMALIZED PRICES $p \cdot \overline{\omega} = p \cdot \int \omega_i \mu(di) = 1$, AND COMPETITIVE EQUILIBRIUM IS UNIQUE

3. EXTENSION FOR PRODUCTION ECONOMIES IF DISTRIBUTIONS OF TASTES AND OF INCOMES (ENDOWMENTS PLUS PROFIT SHARES) ARE INDEPENDENT

→ **APPLICATION**: COURNOT OLIGOPOLY

(GRANDMONT, 1993)

n FIRMS, PRODUCTION y_j, COST $c_j y_j$, $j=1,\ldots,n$

AGGREGATE DEMAND $Q(p)$, INVERSE $P(y)$

REVENUE FUNCTIONS

$$R(y_j, z_j) = y_j P(y_j + z_j), \quad z_j = \sum_{i \neq j} y_i$$

COURNOT EQUILIBRIUM: $y_j^* \geq 0$, $j=1,\ldots,n$ S.T.
y_j^* MAX$_{y_j}$ $R(y_j, z_j^*) - c_j y_j$ GIVEN z_j^*

$\exists$? NEEDS CONCAVE REVENUE FUNCTIONS $R(\cdot, z_j)$. NO GUARANTEE OF THIS PROPERTY WHEN DISTRIBUTION OF INDIVIDUAL CHARACTERISTICS IS ARBITRARY (MACROECONOMIC INDETERMINACY)

"DISPERSION OF TASTES" $\Rightarrow$ CONCAVE REVENUE FUNCTIONS?

AGGREGATE DEMAND $Q(p)=\int e^{\alpha} q(e^{\alpha}p)\, g(\alpha)\, d\alpha$

OR $pQ(p)=\int w(e^{\alpha}p)\, g(\alpha)\, d\alpha$, $w(p)=pq(p)\leq b$

$$\Rightarrow \frac{d[pQ(p)]}{d\operatorname{Log} p} = \int \partial_{\alpha}\left(w(e^{\alpha}p)\right) g(\alpha)\, d\alpha$$

$$= -\int w(e^{\alpha}p)\, g'(\alpha)\, d\alpha$$

$$\left|\quad\right| \leq b\int |g'(\alpha)|\, d\alpha \leq b\, m_1$$

$$\Rightarrow \frac{d^2[pQ(p)]}{(d\operatorname{Log} p)^2} = \int w(e^{\alpha}p)\, g''(\alpha)\, d\alpha$$

$$\left|\quad\right| \leq b\int |g''(\alpha)|\, d\alpha \leq b\, m_2$$

⇒ "DISPERSION OF TASTES" ("FLAT" DENSITY $g(\alpha)$, I.E. m_1 AND m_2 SMALL) IMPLIES THAT AGGREGATE DEMAND BEHAVES (UP TO 2ND ORDER DERIVATIVES) SIMILARLY TO A/p (ELASTICITIES OF $Q(p)$ AND $Q'(p)$, OR OF INVERSE DEMAND $P(q)$ AND $P'(q)$, ARE CLOSE TO -1 AND -2 RESPECTIVELY)

⇒ CONCAVE REVENUE FUNCTIONS AND PROFIT FUNCTIONS $R(y_j, z_j) - c_j y_j$,

⇒ EXISTENCE AND UNICITY OF COURNOT OLIGOPOLY EQUILIBRIUM ($n \geq 2$)

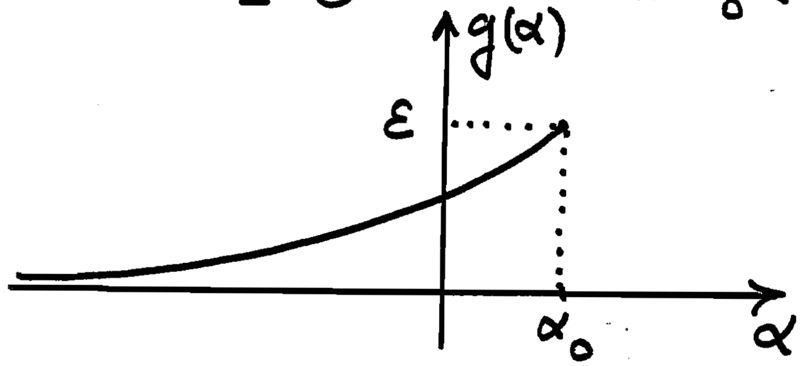
HETEROGENEITY AND CONSTANT ELASTICITY AGGREGATE DEMAND?

AGGREGATE EXPENDITURE

$$W(p) = \int w(e^{\alpha} p)\, g(\alpha)\, d\alpha \,, \qquad w(p) \leqq b$$

DENSITY $g(\alpha) = \varepsilon\, e^{\varepsilon(\alpha - \alpha_0)}$ FOR $\alpha \leqq \alpha_0$

$= 0$ FOR $\alpha_0 < \alpha$



$$\frac{dW}{d\log p}(p) = -\int_{-\infty}^{\alpha_0} w(e^{\alpha} p)\, g'(\alpha)\, d\alpha + w(e^{\alpha_0} p)\, g(\alpha_0)$$

$$\frac{dW}{d\log p}(p) + \varepsilon W(p) = w(e^{\alpha_0} p)\, \varepsilon$$

$\longrightarrow 0$ WHEN $\alpha_0 \rightarrow +\infty$ IF $\lim_{p \to \infty} w(p) = 0$