"NEGISHI-SOLOW EFFICIENCY WAGES, UNEMPLOYMENT INSURANCE AND STOCHASTIC ENDOGENOUS UNEMPLOYMENT BUSINESS CYCLES"

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- ISSUE: DYNAMIC LOCAL INDETERMINACY ($\Rightarrow$ ENDOGENOUS FLUCTUATIONS) OCCURS USUALLY FOR TOO LOW ELASTICITIES OF CAPITAL-LABOUR SUBSTITUTION WHEN PERFECT COMPETITION AND CONSTANT RETURNS

- WAY OUT IN LITERATURE: EXTERNAL INCREASING RETURNS BUT OFTEN TOO LARGE (BENHABIB AND FARMER (1994), CAZZAVILLAN, LLOYD-BRAGA AND PINTUS (1998), BARINCI AND CHERON (2001), HINTERMAIER (2003))

- PAPER:

  - FINANCE CONSTRAINED ECONOMY WITH HETEROGENEOUS AGENTS AND CONSTANT RETURNS AS IN WOODFORD (1986), GRANDMONT, PINTUS AND DE VILDER (1998)

- SHIRKING AND EFFICIENCY WAGES TO INDUCE WORKERS TO MAKE EFFORT LEVEL x REQUESTED IN WAGE CONTRACT (w, x) (NEGISHI (1979), SOLOW (1979), SHAPIRO AND STIGLITZ (1984), DANTHINE AND DONALDSON (1990), UHLIG AND XU (1996), COIMBRA (1999), NAKAJIMA (2006), ALOI AND LLOYD-BRAGA (2006))
- UNEMPLOYMENT INSURANCE (DUFOURT, LLOYD-BRAGA AND MODESTO (2004))

• CONCLUSIONS:

- LOCAL INDETERMINACY ($\Rightarrow$ STOCHASTIC ENDOGENOUS UNEMPLOYMENT FLUCTUATIONS) OCCURS FOR CAPITAL LABOR SUBSTITUTION ELASTICITIES LARGER THAN A VERY SMALL LOWER BOUND, FOR SIGNIFICANT UNEMPLOYMENT INSURANCE
- RAISING UNEMPLOYMENT INSURANCE MAY BE GOOD FOR EMPLOYMENT AND WELFARE ON AVERAGE, ALONG DETERMINISTIC STATIONARY STATE
- CYCLICAL PROPERTIES $\Rightarrow$ POSSIBLE PHILLIPS CURVE?

2. AGENTS BEHAVIOR

- **FIRMS** (CONTINUUM, SIZE 1)

- CONSTANT RETURNS PRODUCTION FUNCTION
$Y_t = AF(k_{t-1}, \ell_t)$ WITH "EFFICIENT" LABOUR
$\ell_t = n_t x_t \geqq 0$, $n_t \geqq 0$ EMPLOYMENT,
$x_t \geqq 0$ EFFORT, OR $Y_t / \ell_t = AF(a_t, 1) = A f(a_t)$
WITH $a_t = k_{t-1} / \ell_t$

- PROFIT MAXIMIZATION

$$\text{MAX} \quad AF(k_{t-1}, \ell_t) - \frac{\omega_t}{x_t} \ell_t - \rho_t k_{t-1}$$

WITH RESPECT TO k_{t-1}, n_t, x_t, ω_t (REAL WAGE W_t/P_t), GIVEN ρ_t (REAL RENTAL RATE OF CAPITAL)

$\Rightarrow$ MARGINAL PRODUCTIVITIES = INPUT COSTS

$$\left| \begin{array}{l} A\rho(a_t) = A f'(a_t) = \rho_t \\ A\omega(a_t) = A(f(a_t) - a_t f'(a_t)) = \omega_t / x_t \end{array} \right.$$

$\Rightarrow$ MINIMIZE ω_t / x_t, REAL WAGE PER UNIT OF EFFORT, UNDER WORKERS' INCENTIVE CONSTRAINT

• CAPITALISTS (CONTINUUM, SIZE 1)

INTERTEMPORAL UTILITY $E_t(\beta_c^{t+j} \operatorname{Log} c_{t+j})$

MORE PATIENT THAN WORKERS $\Rightarrow$ HOLD CAPITAL STOCK k_t AND NO MONEY

$$\boxed{k_t = \beta_c R(a_t) k_{t-1}}$$

WHERE $R_t = R(a_t) = A\rho(a_t) + 1 - \delta =$ GROSS REAL RATE OF RETURN ON CAPITAL

• WORKERS (CONTINUUM, SIZE 1)

- AT t EITHER EMPLOYED (PROBA $0 \leq n_t \leq 1$) OR UNEMPLOYED (PROBA $1 - n_t$)
- FINANCE CONSTRAINT : LABOUR INCOMES (WAGE w_t OR UNEMPLOYMENT INSURANCE νw_t, $0 \leq \nu < 1$) PAID IN CASH AT END OF PERIOD, NO BORROWING.
- IF MORE IMPATIENT THAN CAPITALISTS, $\beta_w < \beta_c$, NOT TOO RISK AVERSE, AND SIGNIFICANT INSURANCE (ν CLOSE TO 1) $\Longrightarrow$ HOLD CASH ONLY

$$u'(c_{t,w}) > \beta_w E_t\left[R_{t+1} u'(c_{t+1,w})\right]$$

- UNEMPLOYMENT INSURANCE FINANCED BY TAXATION OF LABOR INCOMES $\Rightarrow$ IN EQUILIBRIUM (CONSTANT MONEY STOCK M)

$$\boxed{\frac{M}{p_t} = n_t \omega_t = \ell_t \frac{\omega_t}{x_t}}$$

- <u>WORKERS INCENTIVES (NSC)</u>

IF EMPLOYED WORKER WITH WAGE CONTRACT (w_t, x_t) AT t SHIRKS $(x=0)$ AND GETS CAUGHT (PROBA $0<\theta<1$) $\Rightarrow$ FIRED AND GETS νw_t (INSURANCE)

(NSC) DISUTILITY OF EFFORT IS OUTWEIGHTED BY EXPECTED UTILITY GAIN OF NOT SHIRKING

$$V(x_t) - V(0) \leq \theta\, E_t\left[U(c_{t+1,w}) - U(\nu c_{t+1,w})\right]$$

WHERE $c_{t+1,w} = d_t \omega_t p_t / p_{t+1}$ AND $1-d_t$ IS TAX RATE ON LABOR INCOMES (WAGE OR INSURANCE)

3. EFFICIENCY WAGE CONTRACTS

AT t, GIVEN (RATIONAL) ANTICIPATION OF R.V. p_{t+1}, LET $c_{t+1,w} = \bar{c}_{t+1,w}(1+\varepsilon_{t+1})$ WHERE $\bar{c}_{t+1,w} = E_t[c_{t+1,w}]$ OR $E_t[\varepsilon_{t+1}] = \dfrac{1/p_{t+1} - E_t[1/p_{t+1}]}{E_t[1/p_{t+1}]}$

<u>FIRMS</u>: AT t, GIVEN p_t, d_t, u, STOCHASTIC (SUNSPOT) PERTURBATIONS ε_{t+1}, MINIMIZING REAL WAGE PER UNIT OF EFFORT ω_t/x_t GENERATES EFFORT x_t AND EXPECTED CONSUMPTION LEVEL $\bar{c}_{t+1,w}$ THAT MAXIMIZE $x_t/\bar{c}_{t+1,w}$ SUBJECT TO

$$(\text{NSS})\quad V(x_t) - V(0) = \theta E_t\left[U(\bar{c}_{t+1,w}(1+\varepsilon_{t+1})) - U(\bar{c}_{t+1,w}(1+\varepsilon_{t+1}))\right]$$

$$= \Phi(\bar{c}_{t+1,w}, u; \tilde{\varepsilon}_{t+1})$$

<u>- ASSUMPTIONS:</u>

- $V(0) = 0$, FIXED COST OF EFFORT $0 < \lim_{x \downarrow 0} V(x)$
  $V(x) \nearrow$ STRICTLY CONVEX
- $U(c) \nearrow$ STRICTLY CONCAVE

  $c\,U'(c)$ AND $-c^2 U''(c) \nearrow$

  $(\Phi'_{\bar{c}} > 0)$ $\quad$ $(\Phi''_{\bar{c}^2} < 0)$

$\Rightarrow$ AT t, GIVEN DISTRIBUTION OF RANDOM (SUNSPOT) PERTURBATIONS ε_{t+1}, $\exists$ UNIQUE EFFICIENCY WAGE CONTRACT $(x_t, \bar{c}_{t+1,w}) > 0$ SOLVING

$$V(x_t) = \Phi(\bar{c}_{t+1,w}, \nu; \tilde{\varepsilon}_{t+1}), \quad x_t V'(x_t) = \bar{c}_{t+1,w} \Phi'_{\bar{c}}(\bar{c}_{t+1,w}, \nu; \tilde{\varepsilon}_{t+1})$$

($\Rightarrow x_t = x^*$, $\bar{c}_{t+1,w} = c^* \equiv c_{t+1,w}$ CONSTANT OVERTIME WHEN $\tilde{\varepsilon}_{t+1} \equiv 0$, DETERMINISTIC CASE)

($\Rightarrow$ NO ONE SHIRKS IN EQUILIBRIUM)

<u>COMPARATIVE STATICS</u>

– GIVEN $\tilde{\varepsilon}_{t+1}$, <u>INCREASING</u> THE RATE $0 < \nu < 1$ OF <u>UNEMPLOYMENT INSURANCE</u> MAKES

1) $x_t / \bar{c}_{t+1,w} \searrow$, $\bar{c}_{t+1,w} \nearrow$

2) $x_t \rightarrow$ (CONSTANT) IF $R_u(c) = -\dfrac{c\, u''(c)}{u'(c)} \rightarrow$ (CRRA)

3) FOR SMALL $\tilde{\varepsilon}_{t+1}$: $x_t \nearrow$ IF $R_u(c) \searrow$ (DRRA)

COMPARATIVE STATICS (CTN'D)

FOR SMALL $\tilde{\varepsilon}_{t+1}$, INCREASING RISK (SMALL $v_{\varepsilon_{t+1}} = Var_t[\varepsilon_{t+1}] \nearrow$) MAKES

1) $x_t / \bar{c}_{t+1,w} \searrow$

2) $\bar{c}_{t+1,w} \nearrow$ IF $x V''(x)/V'(x) \geqq 1$ AND $c^3 U'''(c) \nearrow$

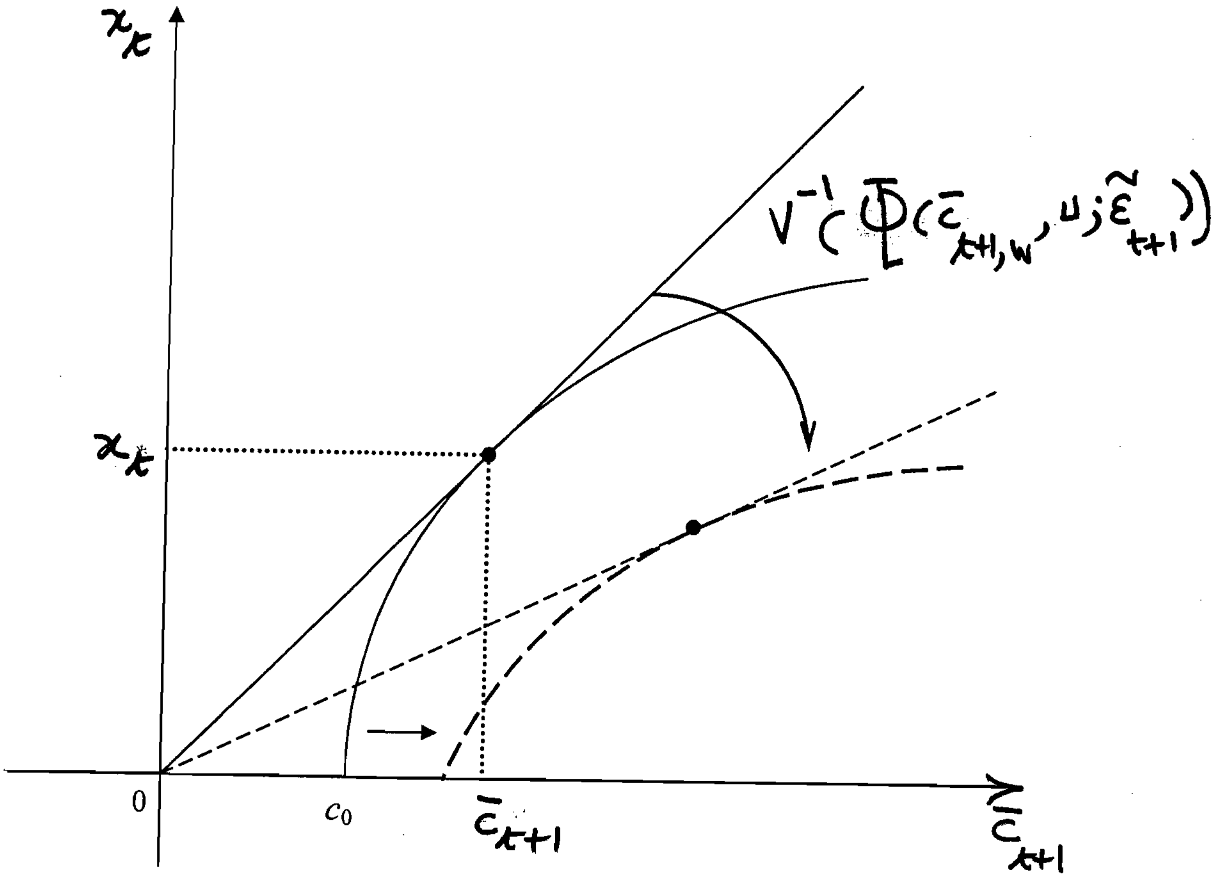
3) $x_t \to$ (CONSTANT) IF $R_u(c) \to$ (CRRA)

4) $x_t \searrow$ IF $c R_u'(c) c U'(c) \nearrow$

(IF DRRA, $R_u'(c) < 0$, $-R_u'(c)$ MUST DECREASE FAST ENOUGH, $R_u(c)$ STRONGLY CONVEX)

FOR HARA FAMILY, $R_u(c) = \dfrac{c}{\chi + \eta c} > 0$

$\Rightarrow$ ABOVE PROPERTIES SATISFIED IF $\chi < 0$ SMALL AND $\eta > 0$ LARGE ($R_u(c) \searrow$ BUT SMALL PERTURBATION FROM CRRA SPECIFICATION $R_u(c) \equiv 1/\eta$ $(\chi = 0)$ WITH LOW RISK AVERSION ($\eta > 0$ LARGE))



x_t
$V^{-1}(\Phi(\bar{c}_{t+1,w}, u; \tilde{\varepsilon}_{t+1}))$
x_t
0
c_0
$\bar{c}_{t+1}$
$\tilde{c}_{t+1}$

Fig. 1

4. INTERTEMPORAL EQUILIBRIA

– CAPITALISTS

$$\boxed{(1) \quad k_t = \beta_c R(a_t) k_{t-1}}$$

WITH $a_t = k_{t-1}/\ell_t$ WITH $R(a_t) = A\rho(a_t) + 1 - \delta$ GROSS REAL RATE OF RETURN ON CAPITAL, AND EFFICIENT LABOR $\ell_t = n_t x_t$

– WORKERS

$$\frac{M}{p_{t+1}} = n_{t+1}\omega_{t+1} = \ell_{t+1}\frac{\omega_{t+1}}{x_{t+1}} = k_t A\, \omega(a_{t+1})/a_{t+1}$$

$$\frac{M}{p_{t+1}} = \left(n_t \bar{c}_{t+1,w} + (1-n_t)\nu \bar{c}_{t+1,w}\right)(1+\varepsilon_{t+1})$$

$$\boxed{\begin{aligned}(2)\quad k_t A\,\omega(a_{t+1})/a_{t+1} &= \left(\frac{\ell_t}{x_t}\bar{c}_{t+1,w} + \left(1-\frac{\ell_t}{x_t}\right)\nu\bar{c}_{t+1,w}\right)(1+\varepsilon_{t+1})\\ &= g\left(\frac{k_{t-1}}{a_t}; x_t, \bar{c}_{t+1,w}\right)(1+\varepsilon_{t+1})\end{aligned}}$$

– SUNSPOT EQUILIBRIA: GIVEN STOCHASTIC PROCESS OF SMALL RANDOM PERTURBATIONS $\{\varepsilon_t\}$ WITH $E_t[\varepsilon_{t+1}] = 0$, SEQUENCE OF R.V. $\{x_t, \bar{c}_{t+1}\}$ DETERMINED BY EFFICIENCY WAGES CONTRACTS.

$\Rightarrow$ (STOCHASTIC) DYNAMICS ON $(k_{t-1}, a_t) \rightarrow (k_t, a_{t+1})$ DEFINED BY (1)(2)

- SUNSPOT STOCHASTIC DYNAMICS = SMALL PERTURBATION OF PERFECT FORESIGHT DETERMINISTIC DYNAMICS OBTAINED BY SETTING $\varepsilon_t \equiv 0$ AND $x_t = x^*$, $\bar{c}_{t+1,w} = c^*$ IN (1), (2):

$$(1^*) \quad k_t = \beta_c R(a_t) k_{t-1}$$

$$(2^*) \quad k_t A\omega(a_{t+1})/a_{t+1} = \left(l_t \frac{c^*}{x^*} + 1 - \frac{l_t}{x^*}\right) \nu c^*$$
$$= g(k_{t-1}/a_t)$$

$\Rightarrow$ $\exists$ NON EXPLOSIVE SUNSPOT EQUILIBRIA NEAR LOCALLY INDETERMINATE STATIONARY STATE, CYCLE OR INVARIANT CLOSED CURVE IN DETERMINISTIC DYNAMICS, IF PROCESS OF SUNSPOT PERTURBATIONS IS SMALL ENOUGH

- STATIONARY STATE: $k_t = \bar{k}$, $a_t = \bar{a} = \bar{k}/\bar{\ell}$, $\bar{\ell} = \bar{n} x^* < x^*$

(1) $\bar{k} = \beta_c R(\bar{a}) \bar{k} \Rightarrow \bar{a}$ S.T. $R(\bar{a}) = 1/\beta_c > 1$

(2) $\bar{n} x^* A \omega(\bar{a}) = \bar{n} c^* + (1-\bar{n}) \nu c^*$

$\Rightarrow$ STATIONARY EMPLOYMENT $0 < \bar{n} < 1$ IF TECHNOLOGY IS PRODUCTIVE ENOUGH, IE IF MARGINAL PRODUCTIVITY OF ONE ADDITIONAL UNIT OF LABOUR WHO MAKES EFFORT x^* EXCEEDS EXTRA CONSUMPTION NEEDED TO INDUCE WORKERS NOT TO SHIRK, $A x^* \omega(\bar{a}) > c^*$

$\Rightarrow$ INCREASING UNEMPLOYMENT INSURANCE MAKES STATIONARY EMPLOYMENT $\bar{n}$ TO GO UP AND GENERATES PARETO WELFARE IMPROVEMENT IF $R_u(c) = -c\,u''(c)/u'(c)$ DOES NOT DECREASE TOO FAST

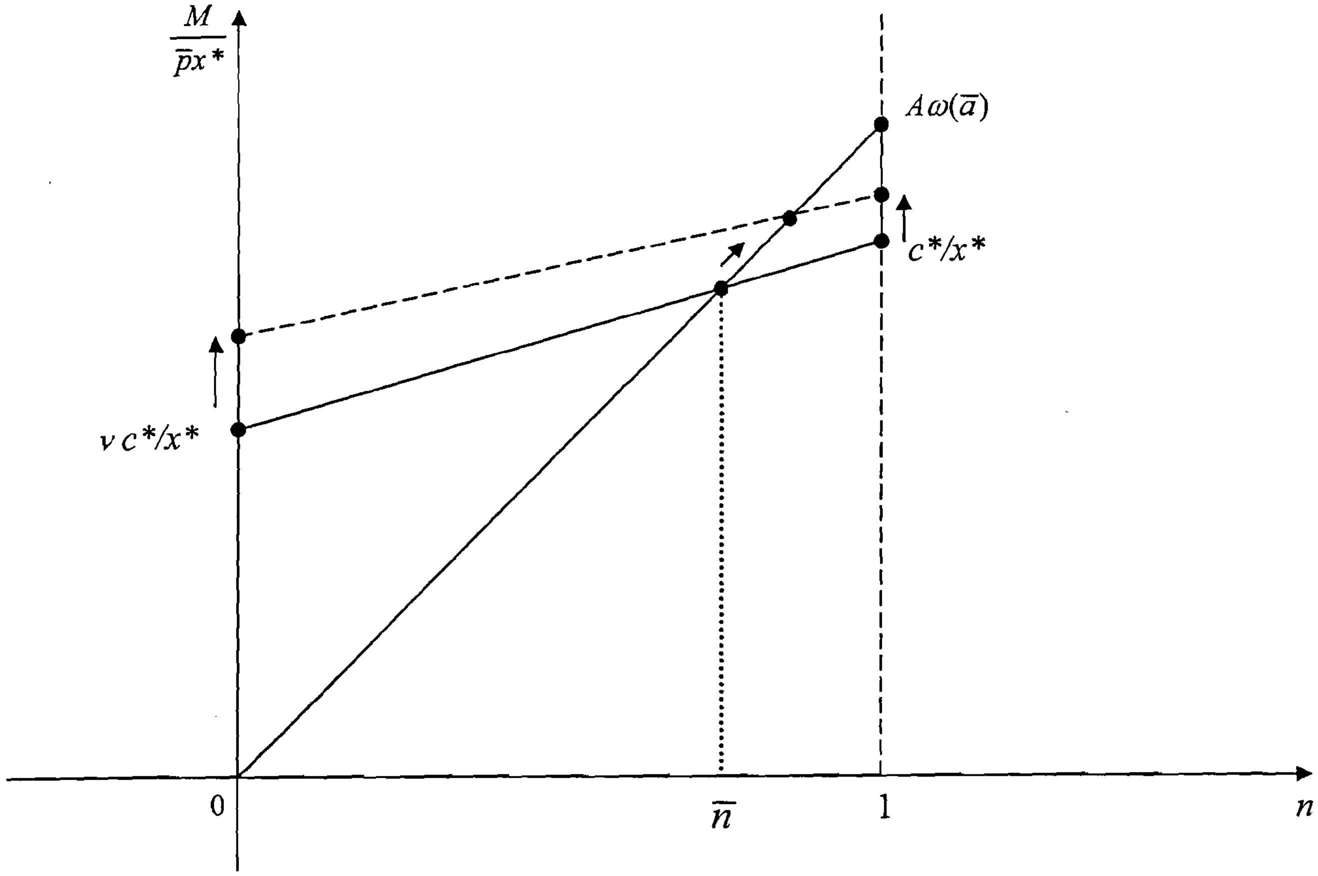


Fig. 2

5. LOCAL INDETERMINACY AND BIFURCATIONS

- PERFECT FORESIGHT DYNAMICS

(1*) $k_t = \beta_c R(a_t) k_{t-1}$

(2*) $k_t A \omega(a_{t+1})/a_{t+1} = g(k_{t-1}/a_t)$

WITH $g(\ell) = \frac{\ell}{x^*} c^* + \left(1 - \frac{\ell}{x^*}\right) \nu x^*$

- SAME DYNAMICS AS IN COMPETITIVE CASE (GPV, 1998) WITH AGGREGATE CONSUMPTION $g(\ell)$ AS A FUNCTION OF EMPLOYMENT $\ell = n x^*$ REPLACING HERE OFFER CURVE $c = \gamma(\ell)$ THERE

- LOCAL ANALYSIS NEAR STATIONARY STATE THROUGH STUDY OF VARIATIONS OF SUM T AND PRODUCT D OF LOCAL EIGENVALUES IN FUNCTION OF ECONOMIC PARAMETERS : SHARE $s = \bar{a}\rho(\bar{a})/f(\bar{a})$ OF CAPITAL IN TOTAL INCOME, ELASTICITY σ OF CAPITAL - EFFICIENT LABOUR SUBSTITUTION, ELASTICITY $\varepsilon_g(\bar{\ell}) = \bar{\ell} g'(\bar{\ell})/g(\bar{\ell})$

→ GEOMETRICAL METHOD:

- CHARACTERISTIC POLYNOMIAL

$$Q(z) \equiv z^2 - Tz + D = 0$$

WHERE T= SUM , D= PRODUCT OF LOCAL EIGENVALUES

- IN (T, D) PLANE:

LINE (AC), $Q(1) = 1 - T + D = 0$

LINE (AB), $Q(-1) = 1 + T + D = 0$

SEGMENT [BC], $D = 1$, $|T| \leq 2$ CORRESPONDS TO PAIR OF COMPLEX CONJUGATE EIGENVALUES

- <u>T, D VARY LINEARLY WITH ELASTICITY ε_g</u>

$\Rightarrow$ <u>HALF LINE $\Delta(\sigma)$</u> THAT MOVES IN PLANE WITH ELASTICITY OF CAPITAL-EFFICIENT LABOUR SUBSTITUTION σ

- LOCAL INDETERMINACY OCCURS WHEN $\Delta(\sigma)$ INTERSECTS STABILITY TRIANGLE ABC, LOCAL BIFURCATION WHEN $\Delta(\sigma)$ INTERSECTS LINE (AB) (FLIP), OR OPEN SEGMENT (BC) (HOPF)

– KEY OBSERVATION: $\varepsilon_g(\bar{\ell}) = \dfrac{(1-\nu)\bar{n}}{\nu + (1-\nu)\bar{n}}$

ONE HAS $0 < \varepsilon_g(\bar{\ell}) < 1$ WHEREAS ONE HAD $\varepsilon_\gamma(\bar{\ell}) > 1$ IN COMPETITIVE CASE.

$\Rightarrow$ INDETERMINACY PROPERTIES WILL BE "OPPOSITE" TO THOSE OBTAINED IN COMPETITIVE CASE

$\Rightarrow$ IF STATIONARY EMPLOYMENT NORMALIZED TO FIX VALUE $\bar{n} < 1$ THROUGH ADJUSTMENT OF PRODUCTIVITY PARAMETER A, THE ELASTICITY ε_g IS CLOSE TO 0 WHEN UNEMPLOYMENT INSURANCE ν IS CLOSE TO 1, AND TO 1 WHEN ν IS NEAR 0.

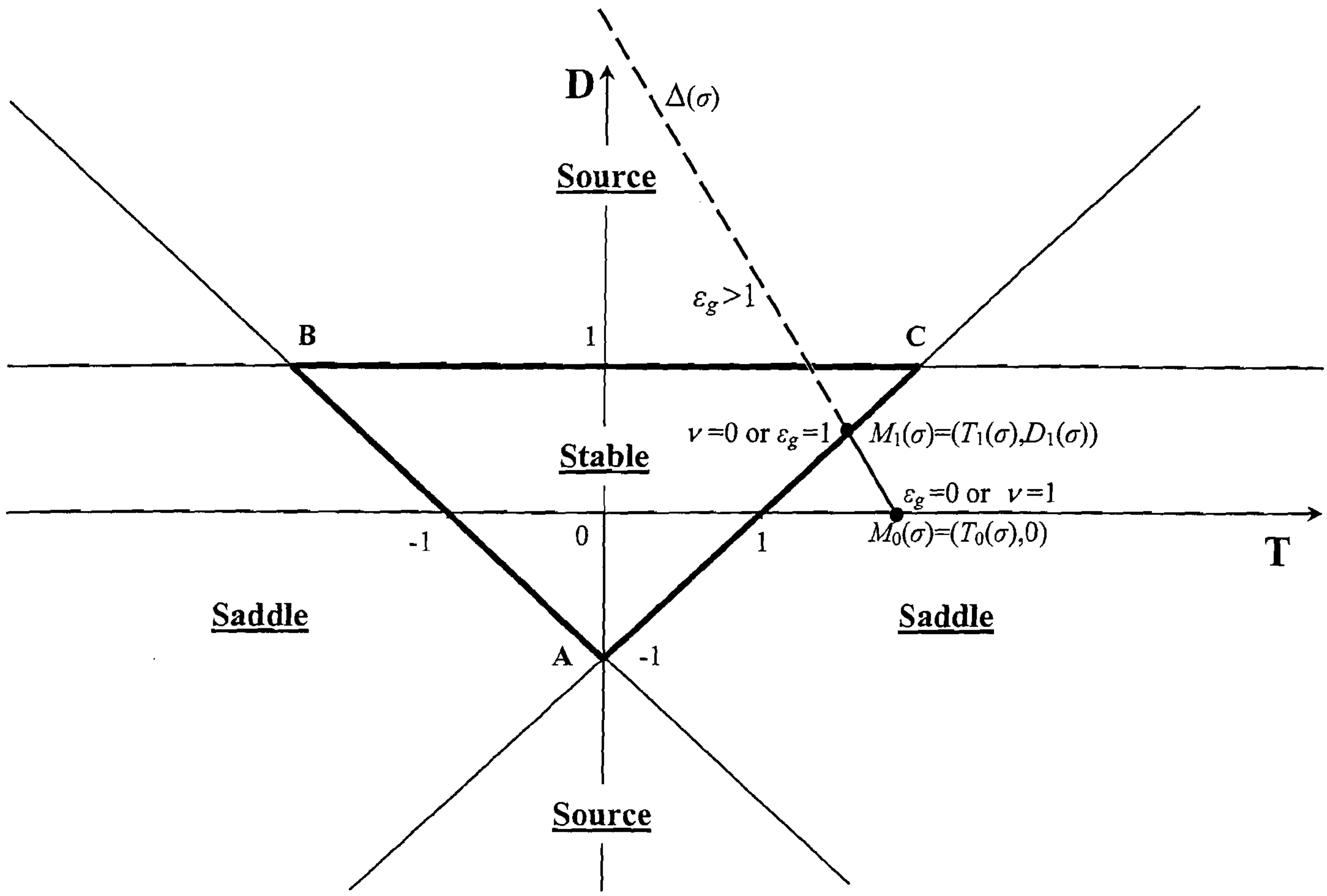

D
Δ(σ)
Source
εg>1
B
1
C
ν=0 or εg=1
M1(σ)=(T1(σ),D1(σ))
Stable
εg=0 or ν=1
-1
0
1
M0(σ)=(T0(σ),0)
T
Saddle
Saddle
A
-1
Source

Fig. 3

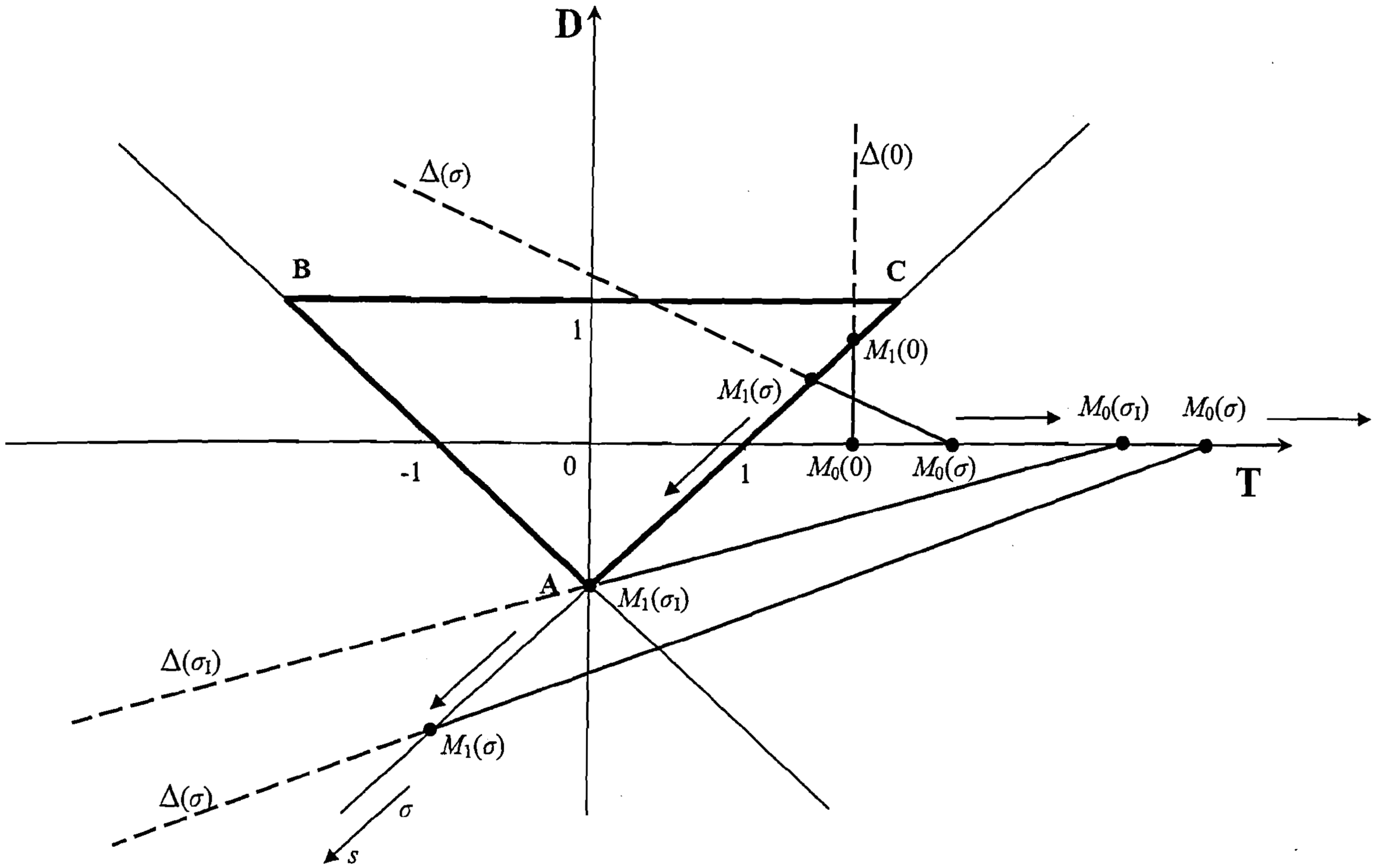


Fig. 4: The case $\sigma < s$

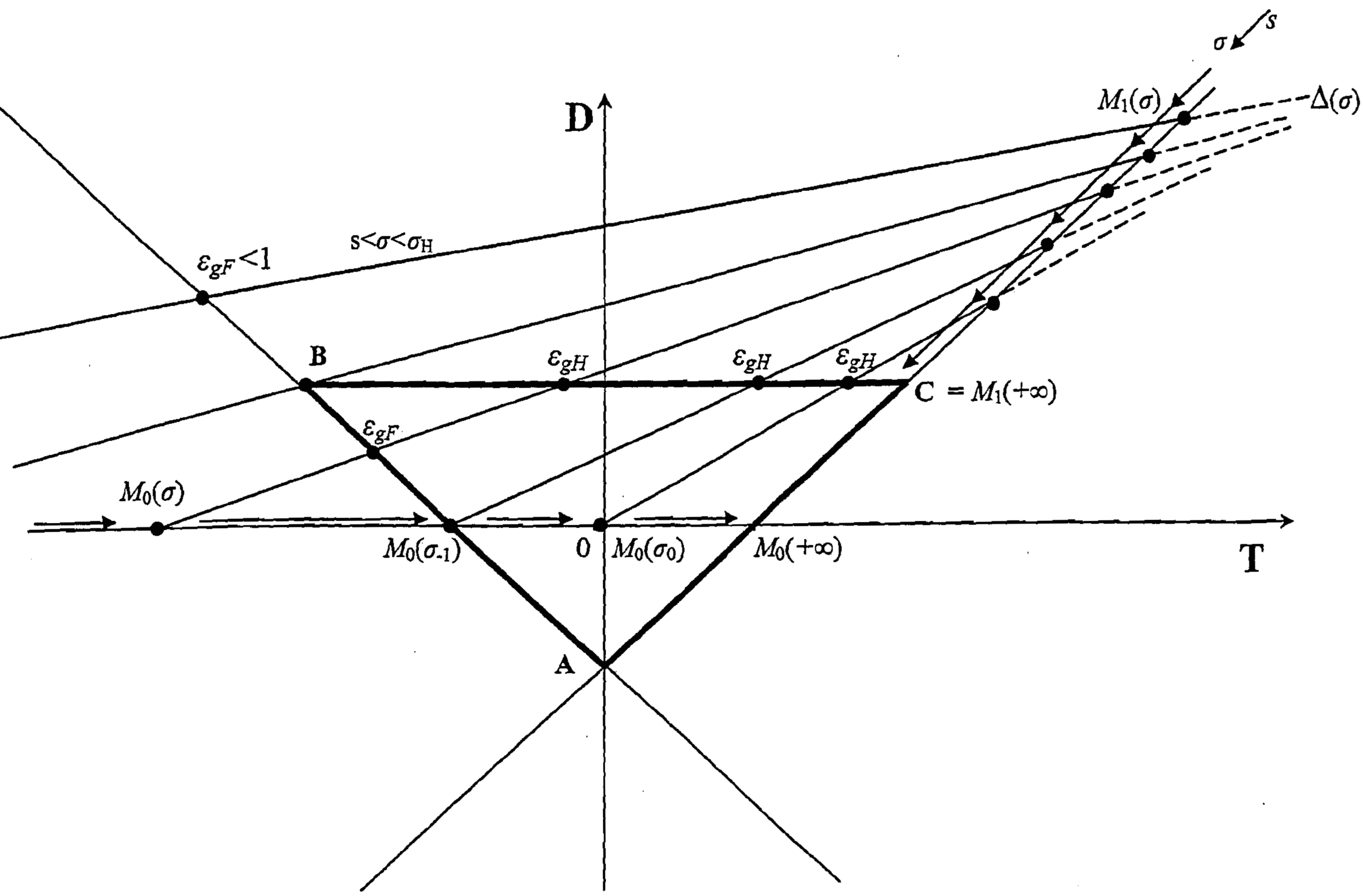


Fig. 5: The case $s < \sigma$

6. CYCLICAL PROPERTIES OF (SMALL) SUNSPOT EQUILIBRIA

ASSUMPTION: $\sigma > s$ (LOCAL INDETERMINACY)

GIVEN PROCESS $\{\varepsilon_t\}$ AND HISTORY UP TO $t-1$, IF $\varepsilon_t \nearrow$, HENCE $c_{t,w} = \bar{c}_{t,w}(1+\varepsilon_t) \nearrow$

$\Rightarrow$:

- $a_t = k_{t-1}/\ell_t \searrow$ HENCE $\ell_t = n_t \lambda_t \nearrow$
- $\omega(a_t) = \omega_t/\lambda_t \searrow$
- CAPITAL $k_t = \beta_c R(a_t) k_{t-1}$ AND OUTPUT $y_t = F(k_{t-1}, \ell_t) \nearrow$
- REAL WAGE ω_t AND EMPLOYMENT n_t ARE PROCYCLICAL ($\nearrow$) IF EFFORT IS MILDLY PROCYCLICAL

$$\frac{s}{\sigma - s} \frac{d\varepsilon_t}{1+\varepsilon_t} < \frac{d\lambda_t}{\lambda_t} < \frac{\sigma}{\sigma - s} \frac{d\varepsilon_t}{1+\varepsilon_t}$$

NEED AUTOCORRELATIONS IN PROCESS $\{\varepsilon_t\}$. IF SUNSPOT PROCESS $\{\varepsilon_t\}$ I.I.D $\Rightarrow \lambda_t$ and $\bar{c}_{t+1,w}$ CONSTANT OVER TIME.

- UNDER ASSUMPTIONS MADE, EFFORT $x_t \nearrow$ WHEN $v_{\varepsilon_{t+1}} = Var[\varepsilon_{t+1} | \varepsilon_t] \searrow$. THUS ABOVE INEQUALITIES VERIFIED (PHILLIPS CURVE) IF ELASTICITY OF $Var[\varepsilon_{t+1} | \varepsilon_t]$ WITH RESPECT IS < 0, SIGNIFICANT BUT NOT TOO LARGE IN ABSOLUTE VALUE

$\Rightarrow$ TAX RATE $1-d_t$ IS COUNTERCYCLICAL $\searrow$

$$\Rightarrow \bar{c}_{t+1,w} = d_t \omega_t E[p_t/p_{t+1} | \varepsilon_t] \searrow$$

$$\Rightarrow E[p_t/p_{t+1} | \varepsilon_t] \searrow$$

- <u>BUT</u> AS FOR MANY CASH IN ADVANCE MODELS, VELOCITY OF CIRCULATION OF MONEY IS CONSTANT $(= 1) \Rightarrow \frac{M}{p_t} = n_t \omega_t \nearrow$: PRICES ARE COUNTER CYCLICAL 

$\Rightarrow$ ALLOWING WORKERS TO HOLD CAPITAL AND MONEY ALLOWS NON CONSTANT VELOCITY OF CIRCULATION OF MONEY

PROMISING AVENUE FOR FUTURE RESEARCH BUT MUCH MORE COMPLEX