Competitive Equilibria of a Large Exchange Economy on the Commodity Space $\ell^\infty$

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The existence of competitive equilibrium for a large exchange economy over the commodity space $\ell^\infty$ will be discussed. Let $\beta > 0$ be a given positive number, and $\ell$ be a positive integer. We will assume that the consumption set $X$ of each consumer is the set of nonnegative vectors whose coordinates after $\ell$ are bounded by $\beta$,

$$X = \{x = (x^t) \in \ell^\infty \mid 0 \leq x^t \text{ for } t \geq 1, \ x^t \leq \beta \text{ for } t > \ell \}.$$

We call the first $\ell$ commodities, $x^1, x^2 \ldots x^\ell$ the primary commodities. We then define the economy as a distribution on the space of consumers’ characteristics following Hart, Hildenbrand and Kohlberg (1974), namely that an economy is a probability measure $\mu$ on the measurable space $(\mathcal{P} \times \Omega, \mathcal{B}(\mathcal{P} \times \Omega))$, where $\mathcal{P}$ is a preference relation on the consumption set $X$ and $\Omega \subset X$ is a space of the endowment vectors. We will denote the economy under consideration by $\mu$. The marginals of $\mu$ will be denoted by subscripts, for instance, the marginal on $\mathcal{P}$ is $\mu_\mathcal{P}$ and so on. The similar definitions on a distribution on $X \times \mathcal{P} \times \Omega$, see the next Definition.

**Definition.** A pair $(p, \nu)$ of a price vector $p \in \ell^1_+$ and a probability measure $\nu$ on $X \times \mathcal{P} \times \Omega$ is called a competitive equilibrium of the economy $\mu$ if the following conditions hold.

(E-1) $\nu_{\mathcal{P} \times \Omega} = \mu$,

(E-2) $\nu(\{(x, \omega, \omega) \in X \times \mathcal{P} \times \Omega \mid px \leq p\omega$ and $x \preceq w$ whenever $pw \leq p\omega\}) = 1$,

(E-3) $\int_X x d\nu_x \leq \int_{\Omega} \omega d\mu_\Omega$.

The main result of this paper reads;

**Theorem 1.** Let $\mu$ be an economy which satisfies the following assumptions (E) and (P).

**Assumption (E).** $\int_{\Omega} \omega d\mu_\Omega \gg 0$,

**Assumption (P).** $\mu_\Omega(\{\omega = (\omega, \omega) \in \Omega \mid \omega > 0\}) = 1$.

Then there exists a competitive equilibrium $(p, \nu)$ for $\mu$. 

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