Competitive Equilibria of a Large Exchange Economy on the Commodity Space ℓ^∞

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The existence of competitive equilibrium for a large exchange economy over the commodity space ℓ^∞ will be discussed. Let $\beta > 0$ be a given positive number, and ℓ be a positive integer. We will assume that the consumption set X of each consumer is the set of nonnegative vectors whose coordinates after ℓ are bounded by β,

$$X = \{\boldsymbol{x} = (x^t) \in \ell^\infty | \ 0 \leq x^t \text{ for } t \geq 1, \ x^t \leq \beta \text{ for } t > \ell\}.$$

We call the first ℓ commodities, $x^1, x^2 \ldots x^\ell$ the primary commodities. We then define the economy as a distribution on the space of consumers' characteristics following Hart, Hildenbrand and Kohlberg (1974), namely that an economy is a probability measure μ on the measurable space $(\mathcal{P}\times\Omega, \mathcal{B}(\mathcal{P}\times\Omega))$, where $\mathcal{P}$ is a preference relation on the consumption set X and $\Omega \subset X$ is a space of the endowment vectors. We will denote the economy under consideration by μ. The marginals of μ will be denoted by subscripts, for instance, the marginal on $\mathcal{P}$ is $\mu_{\mathcal{P}}$ and so on. The similar definitions on a distribution on $X \times \mathcal{P}\times\Omega$, see the next Definition.

Definition. A pair $(\boldsymbol{p}, \nu)$ of a price vector $\boldsymbol{p} \in \ell^1_+$ and a probability measure ν on $X \times \mathcal{P}\times\Omega$ is called a competitive equilibrium of the economy μ if the following conditions hold.

(E-1) $\nu_{\mathcal{P}\times\Omega} = \mu$,

(E-2) $\nu(\{(\boldsymbol{x}, \succsim, \omega) \in X \times \mathcal{P}\times\Omega | \ \boldsymbol{px} \leq \boldsymbol{p}\omega \text{ and } \boldsymbol{x} \succsim \boldsymbol{w} \text{ whenever } \boldsymbol{pw} \leq \boldsymbol{p}\omega\}) = 1$,

(E-3) $\int_X \boldsymbol{x} d\nu_X \leq \int_\Omega \omega d\mu_\Omega$.

The main result of this paper reads;

Theorem 1. Let μ be an economy which satisfies the followinfg assumptions (E) and (P).

Assumption (E). $\int_\Omega \omega d\mu_\Omega \ggg \boldsymbol{0}$,

Assumption (P). $\mu_\Omega(\{\omega = (\omega_\ell, \omega_\infty) \in \Omega | \ \omega_\ell > \boldsymbol{0}\}) = 1$.

Then there exists a competitive equilibrium $(\boldsymbol{p}, \nu)$ for μ.