# A Probabilistic Representation of Exact Games on $\sigma$-Algebras[*]

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## Abstract

The purpose of this paper is to establish the intrinsic relations between the cores of exact games on $\sigma$-algebras and the extensions of exact games to function spaces. Given a probability space, exact functionals are defined on $L^\infty$ as an extension of exact games. To derive a probabilistic representation for exact functionals, we endow them with two probabilistic conditions: law invariance and the Lebesgue property. The representation theorem for exact functionals lays a probabilistic foundation for nonatomic scalar measure games. Based on the notion of $P$-convexity, we also investigate the equivalent conditions for the representation of anonymous convex games.

**JEL Classification:** C71; D81.
**Key words:** Exact game; Core; Exact functional; Choquet integral; Law invariance; Lebesgue property; Anonymity; $P$-convex measure.