# Introduction to Nonadditive Measure Theory

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In 1974, Sugeno introduced the notion of fuzzy measures and integrals to evaluate nonadditive or nonlinear quality in systems engineering. In the same year, Dobrakov independently introduced the notion of submeasures to refine measure theory further. Both fuzzy measures and submeasures are special kinds of nonadditive measures, and their studies have stimulated engineers' and mathematicians' interest in nonadditive refinements of measure theory.

The classical theorems, such as the Egoroff theorem, the Alexandroff theorem, and some convergence theorems of integrals, are fundamental and important to develop measure theory. Therefore, many researchers continue to try to obtain their successful analogues in nonadditive measure theory.

When trying to develop nonadditive measure theory in Riesz spaces, along with the nonadditivity of measures, we confront some tougher problems due to the $\varepsilon$-argument, which is useful in measure theory, but not working well in a general Riesz space. Recently, instead of the $\varepsilon$-argument, we introduce and impose some new smoothness conditions on a Riesz space (the asymptotic Egoroff property, the monotone function continuity property, and so on) to obtain successful analogues of some important results in real-valued nonadditive measure theory.

In this talk, we first distinguish nonadditive measure theory from usual one by illustrating the process of nonadditive generalization of the famous Egoroff theorem about the almost uniform convergence of measurable functions. After that, we will expound on remarkable progress of Riesz space-valued nonadditive measure theory.

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