A SIMPLE AND ELEMENTARY PROOF OF BRROUWER'S FIXED POINT THEOREM

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1. Abstract

The following theorem was proved by Hadamard in 1910. Brouwer gave the other proof of this theorem in 1912. After Brouwer, this theorem is called Brouwer's fixed point theorem. This is the most well known fixed point theorem.

Theorem 1.1 (Hadamard-Brouwer)**.** *Let C be a compact convex subset of an n-dimensional Euclidean space R^n. Let g be a continuous mapping from C into itself. Then, there exists a point $c \in C$ such that $g(c) = c$.*

This result is one of the key theorems characterizing the topology of Euclidean spaces, and used across numerous fields of mathematics. This theorem forms the starting point of a number of more general fixed point theorems. In economics, Brouwer's fixed point theorem and the Kakutani fixed point theorem play a central role in the proof of existence of general equilibrium in market economies.

On the other hand, by the demand of contemporary engineering we are requiring more certain mathematics than conventional mathematics. The Mizar project started around 1973 as an attempt to reconstruct mathematical vernacular in a computer-oriented environment. When the author translates a proof of usual mathematics into the language of Mizar, the author feels that there are many logical gaps in traditional ways of mathematics. The author feels also the mathematical intuition of a human being is a wonderful thing. The author thinks that mathematics in Mizar and usual mathematics are in the complementary relation.

The author thinks that the existing proofs of this theorem are so complex for Mizar. In this talk, the author try to give a really elementary proof of this theorem as a flow chart for Mizar. We need only a few knowledge of Euclidean spaces.

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