REAL IDEAL AND DUALITY RELATED TO POLYNOMIAL OPTIMIZATION

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Polynomial optimization is a problem for finding a minimizer of a polynomial function over a semialgebraic set; a subset of $\mathbb{R}^n$ defined by real polynomial equalities and inequalities. There is an efficient algorithm which solves a sequence of semidefinite programming relaxations and finds a *global minimum* of the original problem under moderate assumptions. The method uses ideas from functional analysis, algebraic geometry and optimization.

In this talk, we discuss sufficient conditions for generated semidefinite programming to have no duality gap. Especially polynomial optimization problems with equality and inequality constraints are considered.

To obtain sufficient conditions, we investigate vanishing ideals of semialgebraic sets in $\mathbb{R}^n$. Let $\mathcal{V}(I) = \{x \in \mathbb{R}^n \mid p(x) = 0, \forall p \in I\}$ for an ideal $I \subset \mathbb{R}[x]$; the variety of I, and $\mathcal{I}(K) = \{p \in \mathbb{R}[x] \mid p(x) = 0, \forall x \in K\}$ for $K \subset \mathbb{R}^n$; the vanishing ideal of K. When we deal with a polynomial ring over $\mathbb{C}$, the Hilbert's Nullstellensatz describes a relationship between varieties and ideals. On the other hand, for an ideal I in $\mathbb{R}[x]$, the Real Nullstellensatz says that $\mathcal{I}(\mathcal{V}(I)) = I$ if I is *real*. some criteria on reality of ideals are reviewed. Then we discuss equivalent conditions for the equality $\mathcal{I}(S \cap \mathcal{V}(I)) = I$, where S is a semialgebraic set. Both conditions $\mathcal{I}(\mathcal{V}(I)) = I$ and $\mathcal{I}(S \cap \mathcal{V}(I)) = I$ are verifiable and closely related to duality of associated semidefinite programming.

References

[1] Yoshiyuki Sekiguchi, Tomoyuki Takenawa, and Hayato Waki, Real ideal and the duality of semidefinite programming for polynomial optimization, preprint: arXiv:0901.2998 [math.OC].

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2000 *Mathematics Subject Classification.* Primary: 14P10 Secondary: 12D15, 14P05, 90C22.

Key words and phrases. real ideal, polynomial optimization, semidefinite programming relaxation, real algebraic geometry.

Topics: Optimization