A New Class of Convex Games on σ-Algebras and the Countable Additivity of the Core[*]

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Abstract

Using a notion of convex combination of measurable sets, we introduce and examine a new kind of convexity, μ-*convexity*, of games defined on a σ-algebra of a nonatomic finite measure space. We show that a μ-convex game is balanced, its core contains a nonatomic finite measure, and if the game is nonnegative and μ-lower semicontinuous, then each element of its core is a nonatomic finite measure. Moreover, we demonstrate that not every μ-convex game is exact. However, if a μ-convex game is exact, then its μ-lower semicontinuity is equivalent to the μ-lower semicontinuity of every element of its core. As an application, we extend the classical result of Dubins and Spanier (1961, "How to cut a cake fairly", *American Mathematical Monthly*, vol. 68, pp. 1–17) regarding the existence of fair division to a certain class of not necessarily additive evaluations.