

A New Class of Convex Games on σ -Algebras and the Countable Additivity of the Core*

Nobusumi Sagara[†]

Faculty of Economics, Hosei University
4342, Aihara, Machida, Tokyo 194-0298, Japan
e-mail: nsagara@hosei.ac.jp

Milan Vlach

School of Mathematics and Physics, Charles University
Malostranské náměstí 25, 118 00 Praha 1, Czech Republic
e-mail: milan.vlach@mff.cuni.cz

May 10, 2008

*This research is part of the International Research Project on Aging (Japan, China and Korea) of the Hosei Institute on Aging, Hosei University, and is supported by a Grant-in-Aid for Scientific Research (No.18610003) from the Ministry of Education, Culture, Sports, Science and Technology.

[†]Corresponding author.

Abstract

Using a notion of convex combination of measurable sets, we introduce and examine a new kind of convexity, μ -convexity, of games defined on a σ -algebra of a nonatomic finite measure space. We show that a μ -convex game is balanced, its core contains a nonatomic finite measure, and if the game is nonnegative and μ -lower semicontinuous, then each element of its core is a nonatomic finite measure. Moreover, we demonstrate that not every μ -convex game is exact. However, if a μ -convex game is exact, then its μ -lower semicontinuity is equivalent to the μ -lower semicontinuity of every element of its core. As an application, we extend the classical result of Dubins and Spanier (1961, “How to cut a cake fairly”, *American Mathematical Monthly*, vol. 68, pp. 1–17) regarding the existence of fair division to a certain class of not necessarily additive evaluations.

Mathematics Subject Classification 2000: Primary: 91A12, 91B32; secondary: 28A10.

OR/MS Subject Classification: Primary: Games/group decision/Non-atomic; secondary: Mathematics/convexity.

Journal of Economic Literature Classification: C71; D63.

Key words: Nonatomic finite measure space; μ -convex games on σ -algebras; Supermodular games; Core; Fair division; Exact games; μ -lower semicontinuity; Countable additivity.