Computation of Cores of Strategic Games with Punishment-Dominance Relations

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Abstract

Games with punishment-dominance relations are strategic form games with non-empty cores. In any game in this class, for any player i and any pair of his/her strategies $\{x_i, y_i\}$, the change of strategy from x_i to y_i makes the others' payoffs unanimously worse off for all the others' fixed strategies or unanimously better off for all the others' fixed strategies: strategy sets are complete with respect to *punishment-dominance relations*. The voluntary contribution game for production of a public good, the n-person prisoners' dilemma game, Cournot's oligopoly model of quantity competition, are all in this class.

The aim of this paper is to propose an algorithm to find a payoff vector in the α-core and the associated α-core strategy for games with punishment-dominance relations. Every computation by the algorithm terminates within $O(|N|^3 \cdot \max\{|X^i|\})$ steps, where not only every elementary operation but also every oracle call to an evaluation of the payoff of a player or the next punishment-dominant strategy is counted as one step.

The idea of this method will be stated in terms of strategic *reduced games*, the α-coalitional form of which are equal to the reduced game defined by Greenberg and Peleg in 1980s, who extended the definition of the traditional TU reduced game to NTU game. By the use of this algorithm, we can find a dominating coalition and its associated imputation in the α-core for any given payoff vector outside the core. In other words, our algorithm prescribes the v-NM stable set.

This result must not be confused with Scarf's algorithm or the supermodular function maximization algorithms. Both of them aims not at strategic form games but at coalitional games, but, in general, it is not easy to

obtain the associated colaitional game (or its characterizing corner vectors) efficiently for any given strategic form game. Even if it can be obtained efficiently, Scarf's algorithm is no efficient because every pivot step of it takes $2^{|N|}$ times steps. Furthermore, the super-modular function maximization algorithms are not applicable for NTU games.