

# Alexander Ioffe 教授(イスラエル工科大学)の来日について

凸解析・変分学の専門家 A. Ioffe 教授が慶應義塾大学経済学部特別招聘教授として来日され、いろいろな形で研究活動に参加されると同時に連続講義もして下さいます。また教授の業績に対し慶應義塾大学は名誉博士の学位を授与することも決まっています。今日までに決まっているスケジュールをご紹介致します。

## 連続講義

慶應義塾大学大学院経済研究科における講義

講義題目	Introduction to Modern Variational Analysis: Stability, Regularity and Critical Points
開催日時	4月5日(水)、12日(水)、19日(水)、26日(水) 水曜日 10時45分～12時15分
	5月8日(月) 月曜日 15時～17時30分
場 所	慶應義塾大学三田キャンパス 水曜日 421 教室 月曜日 456-B 教室

## 講義内容

The course will be devoted to mathematical theories related to two problems that can be loosely formulated as follows:

- (a) what is a regular behavior of a system;
- (b) how the behavior of a system is affected by perturbations (of the system) .

Modern regularity theory, unlike its classical predecessor, is basically quantitative, so the main attention in the lectures will be paid to development, calculation and estimation of quantitative measures of regularity. Most of the results will be accompanied by explanations rather than rigorous proofs. But several proofs which either contain important ideas or are widely applicable within and outside the theory will be given in full details.

## Lecture 1. INTRODUCTION. LINEAR SYSTEMS.

Examples of regular behavior. Banach open mapping theorem (quantitative version).

Perturbations of regular (surjective) linear operators. Fredholm operators. Stability of index.

## Lecture 2. DIFFERENTIABLE MAPPINGS.

Frechet differentiability. Theorems of Lusternik and Graves. Applications: general inverse and implicit functions theorems; exact penalties and necessary optimality conditions in

smooth constraint optimization. Critical values of smooth maps. Sard-Smale theorem. Transversality.

### **Lecture 3. GENERAL THEORY: SET-VALUED MAPPINGS IN METRIC SPACES.**

Basic concepts of regularity theory: openness, Lipschitz stability, metric regularity.

Equivalence theorem. Variational principle of Ekeland. Error bounds Criteria of regularity and estimates for regularity constants. Additive perturbations and radius of regularity. The finite dimensional theory.

### **Lecture 4. CONVEX SET-VALUED MAPPINGS**

Main concepts of convex analysis (separation, Minkowski and Fenchel duality, subdifferential calculus, convex processes, constraint systems). Set-valued mappings with convex graphs, their derivatives and coderivatives. Theorem of Robinson-Ursescu. Calculation of regularity rates. Application: distance to infeasibility; constraint systems of convex and convex semiinfinite programming. Error bounds for convex functions and constraint systems. Stability of solutions and well-posedness of convex optimization problems.

### **Lecture 5. GENERICITY PROPERTIES**

Critical values of set-valued mappings. Small sets. Set-valued mappings with stratified graphs. Basic notions and facts from the theory of o-minimal structures (monotonicity theorem, uniform finiteness theorem, cell decomposition theorem, definable choice theorem). Main examples (semilinear functions, semialgebraic functions, globally sub-analytic functions). Optimization of definable functions: gradient descent and Newton method. Morse-Sard theorem for critical and asymptotically critical values of definable set-valued mappings and set-valued mappings admitting smooth stratification.

#### **連続講義**

#### **慶應義塾大学大学院理工学研究科における講義**

講義題目 Variational Analysis and Non-Smooth Regularity

開催日時 4月27日(木)、4月28日(金)、5月11日(木)

16時30分～18時

場 所 慶應義塾大学矢上台キャンパス

4月27日(木)、5月11日(木)14棟203教室

4月28日(金) 未定

#### **講義内容**

Suppose we have a family of problems depending on a certain parameter. What will happen with solutions when parameter changes? How it is possible to know whether the

behavior of solutions is sufficiently regular in one or another sense or to guarantee such behavior? The classical analysis has developed a variety of tools to deal with such problems: implicit function theorem, Lusternik-Graves theorem, Sard's theorem, Thom's transversality theorem. Modern variational analysis is concerned with extensions of such results and basic ideas behind them to ``bad'' maps and functions. The main stimulus to these studies comes from optimization theory and calculus of variations.

### **Lecture 1. METRIC REGULARITY THEORY.**

Introduction to the subject. Ekeland's variational principle and error bounds. Definition and various characterizations of regular points of mappings between metric spaces.

### **Lecture 2. SUBDIFFERENTIAL CALCULUS AND REGULARITY IN Banach SPACES.**

Brief introduction to the theory of subdifferentials of non-differentiable functions on Banach spaces, subdifferential characterization of regular and critical behavior, non-smooth extensions of the Lusternik-Graves and implicit function theorems.

### **Lecture 3. SARD THEOREM FOR STRATIFIED AND DEFINABLE MAPPINGS.**

A brief account of the theory of definable set in o-minimal structures and a sketch of the proof of an extension of Sard's theorem to non-smooth definable mappings and, more generally, to set-valued mappings whose graphs admit stratifications of a slightly weaker type than Whitney stratification.

### **名誉学位授与式**

平成 18 年 5 月 12 日(金)に下記要領にて開催されます。多数ご出席ください。

学位授与式 慶應義塾大学三田キャンパス演説館にて 14 時 ~

記念講演会 同 旧図書館大会議室にて 14 時 45 分 ~