# The 3rd International Conference on

# Mathematical Analysis in Economic Theory

Date:December 20(Mon.) - 22(Wed.), 2004Venue:Keio University Mita campus, Minato-ku, Tokyo, Japan

Organized by



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Keio Economic Society The 21st Century COE Program at Keio "Integrative Mathematical Sciences" The Oak Society 樫の会

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# Registration Desk: The Lobby of North Building

December 20 (Mon.)	8:45 - 17:10
December 21 (Tue.)	8:45 - 15:30
December 22 (Wed.)	8:45 - 17:10

Hall 1: North Building, Conference Hall (2nd Floor)

Hall 2: East Research Building, Lecture Hall (8th Floor)

# Programme

# December 20 (Mon.), Morning

	renary session. num r	
Chair: Kunio Ka	Chair: Kunio Kawamata (Keio Univ.)	
9:00 - 10:00	Tatsuro Ichiishi (Ohio State Univ.)	
	Cooperative Extensions of the Bayesian Game	
Chair: Toru Mar	Chair: Toru Maruyama (Keio Univ.)	
10:10 - 11:10	Charles Castaing (Univ. of Montpellier)	
	Some Variational Convergence Results for Integral Functionals Using Young	
	Measures	
Chair: Katsuhito Iwai (Univ. of Tokyo)		
11:20 - 12:20	Roy Radner (New York Univ.)	
	A Game-Theoretic Approach to Global Warming	

## **Plenary Session: Hall 1**

# December 20 (Mon.), Afternoon

# Parallel Session: Hall 1

Chair: Mikio Nakayama (Keio Univ.)	
13:30 - 14:30	Joseph Abdou (Universite Paris I)
	From Nakamura Number to Stability Index
14:40 - 15:40	Shigeo Muto (Tokyo Institute of Technology)
	Stable Profit Sharing in Patent Licensing: General Bagaininig Outcomes
Chair: Yukihiko Funaki (Waseda Univ.)	
16:00 - 17:00	Dov Monderer (Israel Institute of Technology)
	Generalized Congestion Games
17:10 - 18:10	Akira Yamazaki (Hitotsubashi Univ.)
	On Blocking Coalitions: Linking Mas-Colell with Grodal-Schmeidler-Vind

## Parallel Session: Hall 2

Chair: Norio Kikuchi (Keio Univ.)		
13:30 - 14:30	Charles A. Stuart (Ecole Polytechnique Federale Lausanne)	
	Differentiability and Bifurcation	
14:40 - 15:40	Hiroshi Matano (Univ. of Tokyo)	
	Traveling Waves in the Presence of Obstacles	
Chair: Toru Mar	Chair: Toru Maruyama (Keio Univ.)	
16:00 - 17:00	Norimichi Hirano (Yokohama National Univ.)	
	Periodic Solutions for Systems of Ordinary Differential Equations	
17:10 - 18:10	Masao Fukushima (Kyoto Univ.)	
	Expected Residual Minimization Method for Stochastic Linear Complementarity	
	Problems	

# December 21 (Tue.), Morning

# Parallel Session: Hall 1

Chair: Yasuo Maeda (Keio Univ.)	
9:00 - 10:00	Nicholas Yannelis (Univ. Illinois)
	Issues Arising in Economies with Asymmetric Information
	(with two other essays)
10:10 - 11:10	Ali Khan (Johns Hopkins Univ.)
	On Choice of Technique in the Robinson-Solow-Srinivasan Model
11:20 - 12:20	Kazuo Nishimura (Kyoto Univ.)
	Indeterminacy in a Dynamic Two-Country Model

# Parallel Session: Hall 2

Chair: Naoto Kunitomo (Univ. of Tokyo)	
9:00 - 10:00	Shigeo Kusuoka (Univ. of Tokyo)
	Homogeneous Law Invariant Coherent Multiperiod Value Measures and their
	Limits
10:10 - 11:10	Freddy Delbaen (Eidgenossische Technische Hochschule Zuerich)
	Hedging bounded claims with bounded outcomes
11:20 - 12:20	Chiaki Hara (Kyoto Univ.)
	Pareto Improvement and Agenda Control of Sequential Financial Innovations

# December 21 (Tue.), Afternoon

Plenary Session: Hall 1		
Chair: Hideo Ka	Chair: Hideo Kanemitsu (Tokyo International Univ.)	
13:30 - 14:30	Leonid Hurwicz (Univ. of Minnesota)	
	Designing Mechanisms for Agents Who Do Not Play Games	
Chair: Hiroshi M	Chair: Hiroshi Matano (Univ. of Tokyo)	
14:40 - 15:40	Alexander Ioffe (Technion-Israel Institute of Technology)	
	On Some Variational Problems Associated with Models of Welfare Economics	
Chair: Ryo Naga	Chair: Ryo Nagata (Waseda Univ.)	
16:00 - 17:00	Jean-Michel Grandmont	
	(Centre de Recherché sur Les Ecoulement, les Surfaces et les Transferts)	
	Fiscally Stable Income Distributions under Majority Voting and Bargaining Sets	

# 17:30 – 19:30 Reception Tsunamachi Mitsui Club

# December 22 (Wed.), Morning

# Parallel Session: Hall 1

Chair: Takao Fujimoto (Kagawa Univ.)	
9:00 - 10:00	Andrew McLennan (Univ. of Minnesota)
	Simple Complexity from Imitation Games
10:10 - 11:10	Steven R. Williams (Univ. of Illinois)
	Limited Observability as a Constraint in Contract Design
11:20 - 12:20	Kazuo Mino (Osaka Univ.)
	Consumption Externalities and Capital Accumulation in an Overlapping
	Generations Economy

# Parallel Session: Hall 2

Chair: Ryozo M	Chair: Ryozo Miura (Hitotsubashi Univ.)	
9:00 - 10:00	Hidetoshi Nakagawa (Tokyo Institute of Technology)	
	Analyses of Mortgage-Backed Securities Based on Unobservable Prepayment	
	Cost Processes	
10:10 - 11:10	Yoshihiko Uchida (Bank of Japan)	
	New Acceleration Schemes with the Asymptotic Expansion in Monte Carlo	
	Simulation	
11:20 - 12:20	Jun Sekine (Osaka Univ.)	
	Explicit Solutions to Optimal Long Term Investment Problems for a CIR Factor	
	Model	

# December 22 (Wed.), Afternoon

# Parallel Session: Hall 1

Chair: Kazuya Kamiya (Univ. of Tokyo)	
13:30 - 14:30	Takashi Kamihigashi (Kobe Univ.)
	Nonlinear Duality for Dynamic Optimization (tentative)
14:40 - 15:40	Bernard Cornet (Universite Paris I & Univ. of Kansas)
	Existence of financial equilibria in a multiperiod stochastic economy

# Parallel Session: Hall 2

Chair: Tadashi Minagawa (Nagoya Univ.)	
13:30 - 14:30	Joaquim Silvestre (Univ. of California)
	The Gain-Loss Asymmetry and Single-Self Preferences
14:40 - 15:40	Makoto Yano (Keio Univ.)
	A Price Competition Game under Free Entry

# **Plenary Session: Hall 1**

Chair: Michihiro Ohyama (Toyo Univ.)	
16:00 - 17:00	Kunio Kawamata (Keio Univ.)
	The Stability of Networks with Direct and Indirect Connections



# Cooperative Extensions of the Bayesian Game<sup>\*</sup> Tatsuro ICHIISHI<sup>†</sup>and Akira YAMAZAKI<sup>‡</sup>

#### Abstract

The first part of this talk provides conceptual discussions of key ingredients. We first present the basic one-shot model of Bayesian society, which synthesizes Harsanyi's Bayesian game and Aumann and Peleg's non-side-payment game (NTU game). Jackson's observation of the equivalence of the two widely adopted formulations of incomplete information is briefly reviewed. Two required meaningful conditions on an endogenously determined strategy are discussed: (1) Radner's measurability condition with respect to an information structure, and (2) Bayesian incentive compatibility. Two descriptive *interim* solution concepts, the Bayesian incentive-compatible coarse core and the Bayesian incentive-compatible *interim* core, are discussed.

The second part presents some results on the two *interim* solutions for the Bayesian pure exchange economy (a specific instance of the Bayesian society) in the private information case. The Bayesian incentive-compatible coarse core is nonempty. The Bayesian incentive-compatible *interim* core may be empty, as pointed out by Yannelis. Sufficient conditions for its nonemptiness are explicitly formulated. It is unlikely that these two positive results are extended to the general framework of Bayesian society.

The materials presented here are taken from: Tatsuro Ichiishi and Akira Yamazaki, *Cooperative Extensions of the Bayesian Game*, World Scientific, forthcoming.

<sup>\*</sup>Presented at the Third International Conference on Mathematical Analysis in Economic Theory, Research Center for Mathematical Economics, Japan, December 20-22, 2004

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# Some variational convergence results for integral functionals using Young measures

C. Castaing<sup>\*</sup> and P. Raynaud de Fitte<sup>†</sup> and A. Salvadori<sup>‡</sup>

The study of variational convergence for integral functionals via the convergence of Young measures with applications to Control problems and some classes of evolution inclusions of second order was developed in a series of papers by Castaing-Raynaud de Fitte-Salvadori. Using the stable convergence and the fiber product for Young measures, we present here a more general study for integral functionals defined on  $L^1_{\mathbb{H}}([0,1],dt) \times \mathcal{Y}([0,1],\mathbb{Y})$  where  $\mathbb{H}$ is a separable Hilbert space,  $\mathbb{Y}$  is a Polish space and  $\mathcal{Y}([0,1],\mathbb{Y})$  is the space of Young measures on  $[0,1] \times \mathbb{Y}$ . In particular, we consider two compact metric spaces  $\mathbb{Y}$  and  $\mathbb{Z}$  and the spaces of Young measures  $\mathcal{H} = \mathcal{Y}([0,1],\mathbb{Y})$ and  $\mathcal{R} = \mathcal{Y}([0,1],\mathbb{Z})$  and we prove that, under suitable hypotheses, the value function

$$U_J(\tau, x) = \sup_{\nu \in \mathcal{R}} \inf_{\lambda \in \mathcal{H}} \{ \int_{\tau}^1 [\int_{\mathbb{Z}} [\int_{\mathbb{Y}} J(t, u_{x,\lambda,\nu}(t), y, z) \,\lambda_t(dy)] \,\nu_t(dz)] \, dt \}$$

is a viscosity subsolution of the Hamilton-Jacobi-Bellman equation  $U_t(t, x) + H(t, x, \nabla U(t, x)) = 0$ . Here  $u_{x,\lambda,\nu}$  is the unique trajectory absolutely continuous solution of the evolution inclusion

$$\begin{cases} \dot{u}_{x,\lambda,\nu}(t) \in -\partial I_{\gamma,Y}(t, u_{x,\lambda,\nu}(t), \lambda_t) \\ + \int_{\mathbb{Z}} [\int_{\mathbb{Y}} g(t, u_{x,\lambda,\nu}(t), y, z) \,\lambda_t(dy)] \,\nu_t(dz), \ a.e. \in [\tau, 1], \\ u_{x,\lambda,\nu}(\tau) = x \in \mathbb{H}, \end{cases}$$

governed by the subdifferential operator  $\partial I_{\gamma,Y}$  associated with the control Young measures  $(\lambda, \nu) \in \mathcal{H} \times \mathcal{R}$ , where J and g are Carathéodory integrands defined on  $[0,1] \times \mathbb{H} \times \mathbb{Y} \times Z$ ,  $\gamma$  is a Carathéodory integrand defined on  $[0,1] \times \mathbb{H} \times \mathbb{Y}$  that is convex on  $\mathbb{H}$ ,  $I_{\gamma,Y}$  is the convex integral functional defined on  $[0,1] \times \mathbb{H} \times \mathcal{M}^1_+(\mathbb{Y})$  by  $I_{\gamma,Y}(t,x,\nu) := \int_Y \gamma(t,x,y)\nu(dy)$  for all  $(t,x,\nu) \in [0,1] \times \mathbb{H} \times \mathcal{M}^1_+(\mathbb{Y})$ , and the integrand H in the HJB equation is

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defined on  $[0,1] \times \mathbb{H} \times \mathbb{H}$  by

$$\begin{aligned} H(t,x,\rho) &= \inf_{\lambda \in \mathcal{M}^{1}_{+}(\mathbb{Y})} \sup_{\nu \in \mathcal{M}^{1}_{+}(\mathbb{Z})} \{ \langle \rho, \int_{\mathbb{Z}} [\int_{\mathbb{Y}} g(t,x,y,z) \,\lambda(dy)] \,\nu(dz) \rangle \\ &+ \int_{\mathbb{Z}} [\int_{\mathbb{Y}} J(t,x,y,z) \,\lambda(dy)] \,\nu(dz) \rangle + \delta^{*}(\rho,-\partial I_{\gamma,\mathbb{Y}}(t,x,\lambda)) \}. \end{aligned}$$

Some limiting properties for nonconvex integral functionals in proximal analysis are also investigated using Komlós convergence. In particular, the following holds. Let  $\mathbb{H}$  be a separable Hilbert space. Let f be a nonnegative normal integrand defined on  $[0,1] \times \mathbb{H}$ . Let  $(u^k)_{k \in \mathbb{N} \cup \{\infty\}}$  be a bounded sequence in  $L^{\infty}_{\mathbb{H}}([0,1])$  say,  $N_{\infty}(u^k) \leq R$  for all  $k \in \mathbb{N} \cup \{\infty\}$  for some R > 0, which converges to  $u^{\infty}$  for the norm  $N_{\infty}$ , with  $u^k \in \text{dom } I_f$  for all  $k \in \mathbb{N} \cup \{\infty\}, (\sigma^k)_{k \in \mathbb{N}}$  a bounded positive sequence in  $L^2_{\mathbb{R}}(([0,1]) \text{ and } (\zeta^k)_{k \in \mathbb{N}})$ be a bounded sequence in  $L^1_{\mathbb{H}}([0,1])$ . Assume that

$$f(t, v(t)) \ge f(t, u^{k}(t)) + \langle \zeta^{k}(t), v(t) - u^{k}(t) \rangle + \sigma^{k}(t) ||v(t) - u^{k}(t)||^{2}$$

for all v in the closed ball  $\overline{B}_{L^{\infty}_{\mathbb{H}}([0,1])}(0,2R)$  and for all  $t \in [0,1]$ . Then there is a filter  $\mathcal{F}$  finer than the Fréchet filter such that

$$\sigma(\mathcal{L}^{\infty}_{\mathbb{H}}([0,1])',\mathcal{L}^{\infty}_{\mathbb{H}}([0,1]) - \lim_{\mathcal{F}} \zeta^{k} = l \in \mathcal{L}^{\infty}_{\mathbb{H}}([0,1])'$$

and

$$\sigma(\mathcal{L}^2_{\mathbb{R}}([0,1]), \mathcal{L}^2_{\mathbb{R}}([0,1]) - \lim_{\mathcal{F}} \sigma^k = \sigma \in \mathcal{L}^2_{\mathbb{R}}([0,1])$$

so that

$$\int_0^1 f(t, v(t)) \, dt \ge \int_0^1 f(t, u^\infty(t)) \, dt + \langle l, v - u^\infty \rangle + \int_0^1 \sigma(t) ||v(t) - u^\infty(t)||^2 \, dt.$$

Consequently,  $l \in \partial^p I_f(u^{\infty})$ . Further, let  $l = l_a + l_s$  be the decomposition of l in absolutely continuous part  $l_a$  and singular part  $l_s$ , then we have

$$\int_{0}^{1} f(t, v(t)) dt \ge \int_{0}^{1} f(t, u^{\infty}(t)) dt + \int_{0}^{1} \langle l_{a}(t), v(t) - u^{\infty}(t) \rangle dt + \int_{0}^{1} \sigma(t) ||v(t) - u^{\infty}(t)||^{2} dt.$$

In particular, assume that f is a lower semicontinuous function defined on  $\mathbb{H}$ ,  $(u^k)$  pointwise converges to  $u^{\infty}$ , and

$$f(x) \ge f(u^{k}(t)) + \langle \zeta^{k}(t), x - u^{k}(t) \rangle + \sigma^{k}(t) ||x - u^{k}(t)||^{2}$$

for all  $k \in \mathbb{N}$ , for all  $x \in \overline{B}_{\mathbb{H}}(0,2R)$  and for all  $t \in [0,1]$ . Then, there is a Lebesgue-negligible set  $\mathcal{N}$  such that for each  $t \in [0,1] \setminus \mathcal{N}$  and each  $x \in \overline{B}_{\mathbb{H}}(0,2R)$ ,

$$f(x) \ge f(u^{\infty}(t)) + \langle \operatorname{bar}(\nu_t), x - u^{\infty}(t) \rangle + \sigma(t) ||x - u^{\infty}(t)||^2,$$

where  $\operatorname{bar}(\nu_t)$  denotes the barycenter of  $\nu_t$ . In particular, the preceding inequality holds for each  $x \in u^{\infty}(t) + \overline{B}_{\mathbb{H}}(0, R)$ , in other words,  $\operatorname{bar}(\nu_t) \in \partial^p f(u^{\infty}(t))$ .

Further applications are presented in a forthcoming paper.

November 18, 2004

## A Game-Theoretic Approach to Global Warming Prajit K. Dutta\* Roy Radner\*\*

## ABSTRACT

Global warming is now recognized as a significant threat to sustainable development on an international scale. We model the global warming process as a dynamic commons game in which the players are countries who can at each date, at a cost, reduce their emissions of greenhouse gases and improve their emissions-producing technologies. The state variables are the current global stock of greenhouse gases and the current energyproducing technologies of the respective countries. The model accommodates exogenously growing populations and capital stocks in each country. For a transnational issue like global warming, the "standard" approaches of mechanism design theory are inadequate in the absence of a world government or equivalent institution for enforcing cooperative agreements. Hence we look for self-enforcing agreements (treaties), i.e., Nash equilibria of the game. We show that there is a large multiplicity of noncooperative equilibria of this game, which creates a role for analysts to discover equilibria that are superior to the status-quo ("business-as-usual") equilibrium. We characterize the business-as-usual equilibrium and the set of global Pareto optimal outcomes. We provide information about the set of equilibria, as well as particular (subgame-perfect) Markov equilibria, and show that if the players' discount factors are not too small, there may be Markov equilibria that sustain outcomes that are Pareto-superior to the business-as-usual equilibrium. Furthermore, if the players' discount factors are sufficiently close to unity, it may be possible to sustain Pareto-optimal (first-best) outcomes.

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# From Nakamura Number to Stability Index (Incomplete and preleminary)

# J. Abdou \*

December 12, 2004

Abstract An interactive form is an abstract model of interaction that generalizes Simple Games and Effectivity Functions. The core, as a solution, as well as the correlated notion of stability can be extended to Interactive Forms. Necessary and sufficient conditions for stability can also be established in this setting, expressed by the absence of cycles. A Stability Index that plays a role similar to that of the Nakamura Number can be defined. This Index measures, loosely speaking, the strategic complexity of a an interactive form. If it is small than it is easy to find in the interacting society a preference profile that prevents the emergence of a solution. To any Strategic Game Form one can associate a special interactive form in such a way that given an equilibrium concept (Nash, strong Nash or others) and a preference profile, solutions of the interactive form are precisely the equilibrim outcomes of the game. As a consequence we have necessary and sufficient conditions for the solvability of the Game Forms. The model allows also a localization of the index in case of instability.

**Keywords:** Interactive Form, Stability Index, Nash Equilibrium, Strong Equilibrium, Solvability, Consistency, Simple Game, Effectivity Function, Acyclicity.

JEL Classification: C70, D71 AMS Classification: 91A44

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# Stable Profit Sharing in Patent Licensing: General Bargaining Outcomes

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May, 2004

#### Abstract

In a generalized framework of oligopolistic markets, we study how many potential licensees an external licensor of a patented innovation should negotiate with on the license issue and how much profit sharing the licensor can gain through the negotiation, from a viewpoint of the stability of coalition structures. The core with coalition structure is empty, unless the grand coalition forms under some condition. The bargaining set with coalition structure is a singleton, if the number of licensees optimal for the licensor is larger than that of non-licensees. The bargaining set coincides with the core, if the core is nonempty.

Keywords : licensing, oligopolistic markets, stable profit sharing, bargaining set with coalition structure, core JEL Classification Numbers : D45, D43, C71

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#### Dov Monderer Technion–Israel Institute of Technology Abstract

#### **Basic definitions**

A Generalized Congestion Form, or in short, a G-congestion form is a tuple  $F = (M, N, (\Sigma_i)_{i \in N}, (w_a)_{a \in M})$  where

- *M* is a finite set of facilities (resources).
- N is a finite set of agents (players).
- $\Sigma_i \subseteq 2^M$  is the set of feasible subsets of  $2^M$  (actions) for *i*. with the restrictions that  $\Sigma_i \neq \emptyset$ , and  $\emptyset \notin \Sigma_i$ .
- $w_a: L_N \to R$  is the facility-payoff function associated with  $a \in M$ , where

$$L_N = \{ (i, S) \in N \times 2^N | i \in S \};$$

If every agent j chooses  $A_j \in \Sigma_j$ , and for  $A = (A_j)_{j \in N}$ ,  $S_a(A) = \{j \in N | a \in A_j\}$ , the total payoff of i is

$$u_i(A) = u_i(A_1, \cdots, A_n) = \sum_{a \in A_i} w_a(i, S_a(A)).$$
 (1)

A congestion form is a G-congestion form for which for every facility  $a \in M$ there exists  $f_a : \{1, \dots, n\} \to R$  such that

$$w_a(i, S) = f_a(|S|), \text{ for every } (i, S) \in L_N.$$

where |S| denotes the number of agents in S. In between congestion forms and generalized congestion forms we define: A congestion form with agent-specific payoffs or in short an AS-congestion form is a G-congestion form such that for every  $a \in M$  there exists a function  $g_a : N \times \{1, \dots, n\} \to R$  such that

$$w_a(i, S) = g_a(i, |S|), \text{ for every } (i, S) \in L_N.$$

Every G-congestion form F uniquely defines a game in strategic form,  $\Gamma_F$ , in which the set of players is N,  $\Sigma_i$  is the set of strategies of i, and the utility functions are given in (1). A game  $\Gamma$  in strategic form is called a *generalized* congestion game, or in short, a *G*-congestion game if  $\Gamma = \Gamma_F$  for some Gcongestion form F. The notions of congestion games and AS-congestion games are analogously defined.

An AS-congestion form is of type q if there exists q|M| functions  $g_a^s$ :  $\{1 \cdots, n\} \to R, a \in M, 1 \leq s \leq q$  (not necessarily distinct), such that for every  $i \in N$  there exists s such that for every  $a \in M$ , and for every  $(i, S) \in L_N$ ,  $w_a(i, S) = g_a^s(|S|)$ . Obviously an AS-congestion form of type q is also of type q + 1. Moreover, every AS-congestion form with n players is of type n. The *index* of an AS-congestion form, q(F) is the minimal integer q for which it is of type q. Obviously  $1 \le q(F) \le n$ . Note that q(F) = 1 if and only if F is a congestion form.

The paper presents three types of results:

1. Characterization Results: Rosenthal (1973) proved that every congestion game is a potential game. Monderer and Shapley (1996) proved that every potential game is isomorphic to a congestion game. We generalize these theorems as follows:

**Theorem 1** Every AS-congestion form of type q induces a q-potential game.

**Theorem 2** Every q-potential game is isomorphic to a AS-congestion game induced by an AS-congestion form of type q.

An important corollary of Theorem 2 is:

**Theorem 3** Every game in strategic form is isomorphic to an AS-congestion game.

**2.** Generalized Congestion Forms and Value Theory: In this section a *solution* is a function that assigns to every cooperative TU game  $v : 2^N \to R$  a vector  $\psi(v) \in \mathbb{R}^N$ .

Let  $F = (M, N, (\Sigma_i)_{i \in N}, (w_a)_{a \in M})$  be a generalized congestion form. We say that F is defined by a solution  $\psi$  if there exists cooperative games  $v_a, a \in M$ such that for every  $i \in N$ , and for every  $S \subseteq N$  with  $i \in S$ ,  $w_a(i, S) = \psi v_a^S(i)$ , where  $v_a^S(T) = v_a(S \cap T)$  for every  $T \subseteq N$ . The following theorem generalizes a result in Monderer and Shapley (1996)

**Theorem 4** Let F be a generalized congestion form, and let  $\psi$  be an efficient solution.  $\Gamma_F$  is a congestion game if and only if  $\psi$  is the Shapley value on  $\{v_a^S \mid a \in M, S \subseteq N\}.$ 

We also characterize semivalues.

**3.** Combinatorial auctions with strategic goods A valuation function defined on a set of goods N can be technically considered as a cooperative game on N. When the goods are, say workers, and there are buyers  $b \in B$  who wish to purchase their services, then in a direct auction mechanism each such buyer submits a valuation function  $v^b$ . In view of previous results I analyze such auctions defined by a solution concept  $\psi$  as follows: Each buyer b submits  $v^b$ , every good chooses a buyer, and every buyer receives the set  $S_b$  of all  $i \in N$  that choose b, and he pays to each  $i \in S_b$ ,  $\psi v^b_{S_b}(i)$ .

# On blocking coalitions: Linking Mas-Colell with Grodal-Schmeidler-Vind\*

Joseph Greenberg<sup>†</sup> Shlomo Weber<sup>‡</sup> Akira Yamazaki<sup>§</sup>

August 2004

This paper was dedicated to the memory of Birgit Grodal, to whom the authors owe an unlimited debt of gratitude. Her wisdom, guidance and friendship is already so sorely missed. Sadly, during the writing of the paper we lost Karl Vind, whose quiet and towering presence can never be replaced.

#### Abstract

In this paper we investigate the question of how many coalitions of a given relative size would block a non-Warlasian allocation in large finite economies. It is shown that in finite economies, if a Pareto optimal allocation is bounded away from being Walrasian, then, for any two numbers  $\alpha, \beta$  between 0 and 1, the proportion of blocking coalitions in the set of all coalitions with relative size between  $\alpha$  and  $\beta$ , is arbitrarily close to  $\frac{1}{2}$ , as the number of individuals in the economy becomes large.

<sup>\*</sup>This paper grew out of the authors' much earlier collaboration when all three were visiting the Department of Economics, University of Bonn during the academic year 1980. The authors thank Werner Hildenbrand for fruitful discussions and ceaseless encouragement.

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# Differentiability and bifurcation

C.A. Stuart

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### Abstract

This lecture concerns the relation between elementary notions of differentiability and bifurcation. It reports on joint work done in collaboration with Gilles Evequoz at ELFL, Lausanne.

Consider a function  $F: X \to Y$  where X and Y are real Banach spaces. The two most common notions of differentiability of F are those named after Gâteaux and Fréchet, but many variants exist. One of these is differentiability in the sense of Hadamard which we express in the following form:  $F: X \to Y$  is Hadamard differentiable at  $u \in X$ 

if there exists a bounded linear operator  $T = F'(0) : X \to Y$  such that

$$\lim_{t_n \to 0} \frac{F(u + t_n v_n) - F(u)}{t_n} = Tv \text{ for all } v \in X$$
  
for all  $\{t_n\} \subset \mathbb{R} \setminus \{0\} \text{ with } t_n \to 0$   
and for all  $\{v_n\} \subset X \text{ with } v_n \to v.$ 

It is easily seen that Fréchet differentiability implies Hadamard differentiability which in turn implies Gâteaux differentiability. But Gâteaux differentiable functions may fail to be Hadamard differentiable and Hadamard differentiable functions may fail to be Fréchet differentiable. However, if  $\dim X < \infty$ , then Hadamard and Fréchet differentiability coincide.

In its simplest form, abstract bifurcation theory deals with equations of the form

$$F(u) = \lambda u$$
 where  $\lambda \in \mathbb{R}, X \subset Y$  and  $F(0) = 0.$  (1)

A point  $\lambda \in \mathbb{R}$  is called a *bifurcation point* for (1) (from the line of trivial solutions  $\mathbb{R} \times \{0\} \subset \mathbb{R} \times X$ ) if there exists a sequence  $\{(\lambda_n, u_n)\} \subset \mathbb{R} \times X$ 

having the following properties

$$F(u_n) = \lambda_n u_n \text{ and } u_n \neq 0 \text{ for all } n \in \mathbb{N},$$
  
$$\lambda_n \to \lambda \text{ and } \|u_n\|_X \to 0 \text{ as } n \to \infty.$$

Let  $B_F \subset \mathbb{R}$  denote the set of all bifurcation points for (1). The most basic result in classical bifurcation theory states that, if F is Fréchet differentiable at u = 0, then  $B_F \subset \sigma(F'(0))$  where

$$\sigma(T) = \{\lambda \in \mathbb{R} : T - \lambda I : X \to Y \text{ is not an isomorphism}\}\$$

denotes the spectrum of a linear operator  $T : X \to Y$ . One then seeks additional assumptions on F and  $\lambda \in \sigma(F'(0))$  that ensure that  $\lambda \in B_F$ . These results have important applications in many fields.

When concrete problems involving differential or functional equations are expressed in the form (1), it is not always the case that F is Fréchet differentiable at 0. Sometimes Hadamard differentiability is the best that can be obtained and, in such cases, we may have  $B_F \not\subset \sigma(F'(0))$ 

In this lecture, a series of basic examples and results concerning these situations will be presented.

## EXISTENCE AND MULTIPLICITY OF PERIODIC SOLUTIONS FOR A SYSTEM OF AUTONOMOUS LOTKA-VOLTERRA TYPE EQUATIONS WITH DELAY

#### NORIMICHI HIRANO

#### 1. Abstract

In this talk, we consider the existence and multiplicity of nonstationary periodic solutions of autonomous delay differential equations of Lotka-Volterra type

(1.1) 
$$\begin{cases} \dot{u}_1(t) = u_1(t)(r_1 - a_{11}u_1(t-\tau) - a_{12}u_2(t-\tau)\cdots - a_{1n}u_n(t-\tau)) \\ \dot{u}_2(t) = u_2(t)(r_2 - a_{21}u_1(t-\tau) - a_{22}u_2(t-\tau)\cdots - a_{2n}u_n(t-\tau)) \\ & \cdots \\ \dot{u}_n(t) = u_n(t)(r_n - a_{n1}u_1(t-\tau) - a_{n2}u_2(t-\tau)\cdots - a_{nn}u_n(t-\tau)) \end{cases}$$

where  $n \geq 1, \tau > 0$ ,  $\{r_i\}_{i=1}^n \subset \mathbb{R}$ ,  $\{a_{ij}\}_{i,j=1}^n \subset \mathbb{R}$  and  $u_i = du_i/dt$  for each  $i = 1, \dots, n$ . A broad class of problems in mathematical biology, economics and mechanics are described in the form above with initial conditions

(1.2) 
$$\begin{cases} u_i(s) = \varphi_i(s), & s \in [-\tau, 0], \quad \varphi_i(0) > 0, \\ \varphi_i \in C([-\tau, 0]) & , & i = 1, 2, \cdots, n. \end{cases}$$

In case n = 1, the problem (1.1) is known as delay logistic equation:

(L) 
$$u = \alpha u(t)(1 - u(t - \tau)),$$

The existence and multiplicity of solutions of delay logistic equation has been investigated by many authors(cf. Hale[2] and Goparlsamy[1]). Our purpose in this talk is to show a new approach to this problem and establish the existence and multiplicity of solutions of problem (1.1). In our approach, we will work on the space of periodic functions instead of considering the initial value problem and we make use of the  $S^1$ -degree theory to show the existence and multiplicity of problem (1.1).

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# Expected Residual Minimization Method for Stochastic Linear Complementarity Problems

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Abstract. This paper presents a new formulation for the stochastic linear complementarity problem (SLCP), which aims at minimizing an expected residual defined by an NCP function. We generate observations by the quasi-Monte Carlo methods and prove that every accumulation point of minimizers of discrete approximation problems is a minimum expected residual solution of the SLCP. We show that a sufficient condition for the existence of a solution to the expected residual minimization (ERM) problem and its discrete approximations is that there is an observation  $\omega^i$  such that the coefficient matrix  $M(\omega^i)$  is an  $R_0$  matrix. Furthermore, we show that, for a class of problems with fixed coefficient matrices, the ERM problem becomes continuously differentiable and can be solved without using discrete approximation. Preliminary numerical results on a refinery production problem indicate that a solution of the new formulation is desirable.

This is a joint work with Xiaojun Chen (Hirosaki University).

## Title of the talk of Nicholas Yannelis

## Issues arising in economies with asymmetric information

**Abstract**: The conceptual difficulties of the Rational Expectations Equilibrium (REE) are pointed out. In particular, it is shown that the REE contacts (trades) need not be incentive compatible need not be Pareto optimal and need not be implementable as a perfect Bayesian equilibrium of an extensive form game tree.

A comparison of the REE with the private core indicates that the private core results in trades that are always incentive compatible, and implementable as a perfect Bayesian equilibrium of an extensive form game. Furthermore, unlike the REE which in general doesn't exist, the private core does exist once concavity and continuity assumptions are imposed.

The lack of the above properties of the REE, suggests that a new, "proper" REE must be introduced which results in trades that exist, are incentive compatible and are also implementable as a perfect Bayesian equilibrium of an extensive form game tree. But, what is the appropriate new REE concept which has the above properties? How it is possible to obtain incentive compatible and fully Pareto optimal allocations? Answers are provided to the above questions.

## Perfect Competition in Differential Information Economies\*

Yeneng  $\operatorname{Sun}^{\dagger}$  and Nicholas C. Yannelis<sup>‡</sup>

November 12, 2004

#### Abstract

The idea of perfect competition for an economy with differential information is formalized via an idiosyncratic signal process in which the private signals of almost every individual agent can influence only a negligible group of agents, and the individual agents' relevant signals are essentially pairwise independent conditioned on the true states of nature. Thus, there is no incentive for an individual agent to manipulate her private information. The existence of incentive compatible, ex post Walrasian allocations is shown for such a perfectly competitive differential information economy with or without "common values". Consequently, the conflict between incentive compatibility and Pareto efficiency is resolved exactly, and its asymptotic version is derived for a sequence of large, but finite private information economies.

**Keywords:** Asymmetric information, perfect competition, Pareto efficiency, Walrasian allocation, incentive compatibility, negligible private information.

 $<sup>^*</sup>$ This work was initiated in October 2003 while Yeneng Sun was visiting the University of Illinois at Urbana-Champaign.

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# Non-implementation of rational expectations as a perfect Bayesian equilibrium \*

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**Summary**. We show that a rational expectations equilibrium need not be incentive compatible, need not be implementable as a perfect Bayesian equilibrium and may not be fully Pareto optimal, unless the utility functions are state independent. A comparison of rational expectations equilibria with core concepts is also provided.

## JEL Classification Numbers: 020, 226.

**Keywords and Phrases:** Differential information economy, Rational expectations equilibrium, Coalitional Bayesian incentive compatibility, Implementation, Game trees, Private core, Weak fine core, Interim weak fine core.

<sup>\*</sup>We wish to thank Dr A. Hadjiprocopis for his invaluable help with the implementation of Latex in a Unix environment.

## On Choice of Technique in the Robinson-Solow-Srinivasan Model\*

M. Ali Khan<sup>†</sup> and Tapan Mitra<sup>‡</sup>

September 2003

Abstract: We report results on the optimal "choice of technique" in a model originally formulated by Robinson, Solow and Srinivasan. By viewing this model as a specific instance of the general theory of intertemporal resource allocation associated with Brock, Gale and McKenzie, we resolve long-standing conjectures in the form of theorems on the existence and price-support of optimal paths, and on their long-run behavior. We also examine policies, due to Stiglitz, as a cornerstone for a theory of transition dynamics in this model. We present examples to show that (i) an optimal program can be periodic, (ii) a Stiglitz' program can be bad, and (iii) a Stiglitz production program can be non-optimal. We then provide sufficient conditions under which the policies proposed by Stiglitz coincide with optimal behavior. (127 words)

Journal of Economic Literature Classification Numbers: D90, C62, O21.

*Key Words:* Choice of technique, overtaking criterion, optimal program, golden-rule stock, golden-rule price, value-loss, average turnpike property, long-run, transition dynamics, Stiglitz program, Stiglitz production program, price-support property, cycling.

Running Title: Choice of Technique

<sup>\*</sup>The work reported here is part of a project with a long gestation period: it was initiated during Mitra's visit to the Department of Economics at the University of Illinois in 1986, received invaluable impetus from Professor Robert Solow's presentation at the Srinivasan Conference held at Yale in March 1998, was continued when Khan visited the Department of Economics at Cornell in November 1998, October 2000 and in July 2003, and the EPGE, Fundação Getilio Vargas in January 2001 and December 2002. The authors are grateful to all of these institutions for their hospitality, to the *Center for a Livable Future* at Johns Hopkins for research support, and to two anonymous referees of IJET and Mr. Chris Metcalf for their careful reading. Khan also thanks Professors Abhijit Banerjee, Jimmy Chan, Avinash Dixit and Debraj Ray for stimulating conversation.

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# Indeterminacy in a dynamic two-country model

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## July 30, 2004

#### Abstract

The purpose of this paper is to show that indeterminacy arises in a simple competitive two-country dynamic model of international trade which is based on the standard assumption of international factor immobility, and free of externalities, imperfect competition, and government intervention. This seemingly surprising result comes from the intrinsic properties of the standard dynamic trade model, a system that includes neither an international credit market nor international factor mobility. As will be shown later, dynamic equilibrium paths of our two-country, therefore heterogeneous consumer, model are not Pareto-optimal.

# Homogeneous Law Invariant Coherent Multiperiod Value Measures and their Limits

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#### Abstract

The concept of coherent risk measures were introduced by Artzner et al., , and a characterization theorem was given by Delbaen. Recently coherent multiperiod risk measures were introduced and many studies have already appeared.

On the other hand, the concept of law invariant coherent risk measures was given. In the present paper, we extend this idea to multiperiod ones. The basic tool is the concept of conditional law invariant coherent risk measures. We remark that such kind of ideas is not new (c.f. Gerber ). We also studies continuous limits of such risk measures.

This work is joint work with Yuji MORIMOTO in Integrated Finance Limited (Japan).

# Hedging bounded claims with bounded outcomes

## Freddy Delbaen<sup>1</sup>

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**Abstract.** We consider a financial market with two or more separate components each driven by a Brownian Motion. We look at the problem to hedge a bounded contingent claim in such a way that all the components remain bounded. The problem can also be rephrased as a problem in risk measures.

**Key words:** Hedging, infimal convolution, coherent utility functions, Fatou property, Brownian Motion

Mathematics Subject Classification (2000): 60G44

 $<sup>^1\</sup>mathrm{The}$  author thanks Credit Suisse for the support. Only the author is responsible for the contents of this paper.

# Pareto Improvement and Agenda Control of Sequential Financial Innovations

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### Abstract

In a model of an exchange economy under uncertainty with two periods, one physical good, and finitely many states of the world, we show that for every (complete or incomplete) market span there exists a sequence of securities such that as they are introduced into markets one by one, the prices of any security is not affected by the subsequent introduction of newer securities. Given the absence of pecuniary externalities, this result implies that every stage of such sequential financial innovations is Pareto improving. An implication of this result on financial innovations via an unanimous voting rule is also explored.

## DESIGNING MECHANISMS FOR AGENTS WHO DO NOT PLAY GAMES

### By Leonid Hurwicz and Stanley Reiter

In this work, a mechanism consists of three elements: a message space, a set of response rules, and an outcome rule. In a broader perspective, response rules govern a dynamic process and may originate from established traditions or legislation.Outcome rules are, at least in part determined by the laws of nature (physics, biology), as well as by tradition and/or legislation. In our static framework, response rules define equilibrium relations.

Since we are interested in mechanism design, the message space as well as the equilibrium and outcome rules (other than those dictated by nature) become the unknowns of the problem.

the environment (i.e., the agents' characteristics) and the mechanism determine the equilibrium outcomes. Our objective is to discover mechanisms satisfying specified desirability criteria of equilibrium outcomes. In a simple framework such a criterion is obtained by specifying a (social) goal function whose range is a space of outcomes, and the domain a class of environments (lists of individual agents' characteristics). A mechanism is said to realize a goal function if, in any given environment, its equilibrium outcomes agree with those specified by the goal function for that environment. It is said to be informationally decentralized if, when presented with a proposed message space point, an agent is able to respond with acceptance or rejection based only on this agent 's knowledge its own characteristic.

Given a goal function, our procedure for constructing a mechanism consists of two phases: (1) defining a covering family of subsets of the parameter space;

(2) choosing a transversal for that family, so that the intersection points of the transversal with the subsets generate a message space. We call our choices in the two phases the Method of Rectangles and the Method of Transversals respectively.

The subsets are so chosen that the mechanism generated is informationally decentralized and realizes the given goal function. It is then shown that our choice of the covering family is informationally efficient in the class of decentralized mechanisms realizing the given goal function. (This means that the covering of the parameter space by the subsets belonging to the family is maximally coarse.) Moreover, although not every choice of an acceptable family yields a message space of size (cardinality, dimension) minimal for the given goal function and the class of admissible environments, there exists a family yielding both a message space of minimal size and maximal coarseness .

When the agents' strategic behavior is ruled out, the problem of constructing a decentralized mechanism realizing a goal function becomes trivial. For instance, direct revelation would do. Implementation (e.g., in Nash equilibria) is a special case of informationally decentralized realization. Since it admits strategic behavior, it narrows the class of eligible mechanisms and hence can be expected to result in loss of informational efficiency

# ON SOME VARIATIONAL PROBLEMS ASSOCIATED WITH MODELS OF WELFARE ECONOMICS

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## Abstract

We shall consider several classes of variational problems associated with certain models of welfare economics with infinitely many (measure space) of agents, in the sense that formalization of the models leads to problems belonging to one or another class. The discussion will be centered around two types of questions:

(a) analysis of the variational problems in the general setting (with no regard to economic interpretation) with the main emphasis to the issues of characterization of solutions, existence of solutions and relaxation, and

(b) analysis of specific features of the data coming from formalization of the corresponding problems of economics with the main emphasis to the assumptions that would make the general results of the first part applicable

## Fiscally Stable Income Distributions under Majority Voting and Bargaining Sets

Jean-Michel GRANDMONT, CNRS-CREST, Paris and ICEF, Venice International University, Italy.

### Abstract

We explore two variants of the Bargaining Set in a simple majority game on income distributions in order to understand the apparent stability of tax schedules in democratic societies, despite the fact that the core of such games is empty (no majority Condorcet winner). Those variants are sharper than in the literature (Mas-Colell (1989), Shitovitz (1989), Zhou (1994)), by requiring that counterobjections try to garantee their initial income levels to all members of the minority who stand to lose in an objection. A ...rst variant de...nes as usual an income disbribution to be stable if there is no objection against it that is "justi...ed", i.e. for which there is no counterobjection satisfying the above requirement. A second variant alllows objecting majorities to look one more step ahead. An objection is "weakly justi...ed" if, whenever there is a counterobjection, the objecting majority can beat it while guaranteeing their income levels to all its members. An income distribution is stongly stable if there is no weakly justi...ed objection against it.

These two variants generate sharper solution sets than when applied to large market games as in Mas-Colell (1989), Shitovitz(1989). An income distribution is stable if and only if its Lorenz curve has no point in common with the graph C of  $f : [1/2, 1] \rightarrow [0, 1]$ , with f(b) = 1 - 1/(2b), for b > 1/2. It is strongly stable if and only if it is the equilibrium one.

JEL Classi...cation numbers : C71, D31, D72, H24

Keywords : Inequality, income distribution, stable tax schedules, majority voting, cooperative games, core, bargaining set.

# Extended Abstract Simple Complexity from Imitation Games

by

Andrew McLennan and Rabee Tourky

Gilboa and Zemel (1989) (henceforth GZ) established that certain computational problems related to Nash equilibrium of two player games are **NP**-hard. Connitzer and Sandholm (2003) (henceforth CS) have recently proved refinements of these results using different methods. In this paper we establish refinements of the results of GZ using imitation games, which are a class of two player game studied in McLennan and Tourky (2004). In comparison with GZ and CS, our arguments are simple and direct.

A decision problem is a computational problem for which the desired answer in either YES or NO. A computational problem is in **NP** if there is a polynomial time procedure that has a positive probability of establishing that the answer is YES when that is the case. A problem P is **NP**-hard if any other **NP** problem Q can be polynomially reduced to P, by which we mean that there is a polynomial time procedure for transforming an input for Q into an input for P with the same answer. A problem is **NP**-complete if it is both in **NP** and **NP**-hard. There are many known **NP**-complete problems, including CLIQUE (Given a simple undirected graph, is there a set of k vertices with an edge between each pair in the set?) and SAT (Given a conjunction of disjunctions of primitive propositions  $P, Q, \ldots, R$  and their negations, is there a vector of truth values for  $P, Q, \ldots, R$  such that the conjunction of disjunctions is true?)

The computational difficulty of computing a Nash equilibrium is unknown, but several related decision problems were shown by GZ to be **NP**-hard, including "Is there a second Nash equilibria?" Among other reductions, GZ obtain their results by giving polynomial reductions of the problems of interest to CLIQUE, while CS reduce to SAT. Their constructions are complex, with incentives that are far from obvious.

An *imitation game* is a two player normal form game in which the two agents' sets of pure strategies are the same and the second agent's payoff is 1 if she plays the same pure strategy as the first agent and 0 otherwise. The study of this class of games was initiated in McLennan and Tourky (2004), where we showed that, in spite of their simplicity, they capture the full computational complexity of general two person games. Here we give imitation games for which the problems studied by GZ and CS reduce, by straightforward arguments, to CLIQUE. In addition to providing simpler proofs of these results, this work lends further support to the contention that imitation games are a useful setting for studying complexity issues related to two person games because such issues tend to have intuitively simple and straightforward representations in this context.

# Limited Observability as a Constraint in Contract Design<sup>\*</sup>

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March 15, 2004

#### Abstract

Limited observability is the assumption that economic agents can only observe a finite amount of information. Given this constraint, contracts among agents are necessarily finite and incomplete in comparison to the ideal complete contract that we model as infinite in detail. We consider the extent that finite contracts can approximate the idealized complete contracts. The objectives of the paper are: (i) to identify properties of agents' preferences that determine whether or not finiteness of contracts causes significant inefficiency; (ii) to evaluate the performance of finite contracts against the ideal optimal contract in a bilateral bargaining model.

**Keywords**: limited observability, bounded rationality, contractual incompleteness

JEL Classification Numbers: D82, C78

<sup>\*</sup>This paper originated in In-Koo Cho's seminar on bounded rationality. We thank In-Koo for inviting us to participate in his seminar and for his comments concerning this paper. We also thank Steven Matthews, Nabil Al-Najjar, Jim Peck, Stan Reiter and Tim Van Zandt for their suggestions.

## Consumption Externalities and Capital Accumulation in an Overlapping Generations Economy

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### Abstract

This paper extends the standard overlapping generations model of capital accumulation by introducing consumption externalities. It is assumed that each generation's felicity depends on the social level of benchmark consumption as well as on its own consumption. Since the benchmark consumption is represented by the average consumption of all agents, the contemporaneous consumption externalities are determined by both intragenerational and intergenerational interactions among the consumers. Given this setting, we show that even in a simple model with a logarithmic utility function, the presence of consumption externalities may significantly affect the dynamic behavior and steady-state characterization of the economy. We also reveal that the same conclusion holds in an endogenous growth setting in which production externalities sustain unbounded growth.

# Analyses of Mortgage-Backed Securities Based on Unobservable Prepayment Cost Processes\*

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Graduate School of International Corporate Strategy Hitotsubashi University 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439 Japan

#### Abstract

We propose a prepayment model of mortgage based on a structural approach in order to analyze a prepayment risk of mortgage-backed securities (MBS).

We introduce a continuous process named prepayment cost process. Specifically, each mortgager's prepayment time is defined by the first time when her or his prepayment cost process falls below zero, but prepayment cost processes are supposed to be unobservable in the market. We also introduce a unique risk to the MBS market, called a loan pool risk (LPR), and we regard LPR as a systematic risk other than interest rate.

Using the model, we discuss the conditional distribution of prepayment times and a risk-neutral valuation of pass-through MBS. It is shown that each mortgager's conditional non-prepayment probability and the posterior distribution of LPR play quite important roles in our study.

**Keywords:** Mortgage-Backed Securities(MBS), prepayment cost, loan pool risk, risk-neutral valuation

<sup>\*</sup>This research is partially supported by Grant-in-Aid for Young Scientists (B) No. 16710108 from the Ministry of Education, Culture, Sports, Science and Technology.

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# New Acceleration Schemes with the Asymptotic Expansion in Monte Carlo Simulation

Akihiko Takahashi<sup>#</sup> Yoshihiko Uchida<sup>\*</sup> Ryusuke Matsuoka<sup>#</sup>

In the present paper, we propose a new computational technique with the Asymptotic Expansion (AE) approach to achieve variance reduction of the Monte-Carlo integration appearing especially in finance. In order to compute control variables, we utilize the analytic approximation based on AE in Takahashi (1999) and Kunitomo and Takahashi (2003a). We extend the algorithm developed by Takahashi and Yoshida (2003) to the second order asymptotics. Moreover, we apply AE to approximate time dependent differentials of the target value in two kinds of Newton (1994)'s schemes.

Through numerical experiments, we observe remarkable acceleration of convergence, which implies broad applications of our techniques. Our numerical examples include pricing of average, basket and swap options and their risk parameters (delta, gamma) when the underlying state variables follow Constant Elasticity of Variance (CEV) processes.

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# Explicit solutions to optimal long term investment problems for a CIR factor model

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November 19, 2004

On a financial market with one riskless bond and one risky stock, both of which are affected by a stochastic factor described as a Bessel process with a linear drift (i.e., the square root of Cox-Ingersol-Ross's stochastic interest rate model), explicit representations of the solutions are given for the optimization problems: (i) power-utility maximization of the terminal wealth, (ii) its risk-sensitive version with infinite time horizon, (iii) a large deviations control problem.

# Nonlinear duality for dynamic optimization (tentative)

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## Abstract

This paper provides a necessary and sufficient condition for optimality in a general dynamic optimization problem. In particular, it is shown that a feasible path is optimal iff there is a sequence of "nonlinear dual variables" that supports it.

# Existence of financial equilibria in a multiperiod stochastic economy

Laura Angeloni<sup>1</sup>, Bernard Cornet<sup>2</sup>

## Abstract

We consider the model of a stochastic financial exchange economy where time and uncertainty are represented by a finite event-tree of length T. We provide a general existence result of financial equilibria, which allows to cover several important cases of financial structures considered in the literature, such as nominal and numeraire assets, when consumers may have constraints on their portfolios.

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"The gain-loss asymmetry and single-self preferences" by Antoni Bosch-Domènech and Joaquim Silvestre

## Abstract

Daniel Kahneman and Amos Tverky argued that decisions under uncertainty display a fundamental asymmetry between gains and losses: many people prefer a sure gain of pz to an uncertain gain of z with probability p, while preferring an uncertain loss of z with probability p to a certain loss of pz: they labeled this phenomenon the *reflection effect*.

The present paper explores the extent to which the reflection effect, understood as occurring at a variety of wealth levels, violates (a) single-self preferences, or (b) the expected utility hypothesis. We find that (1) The reflection effect is compatible with single-self preferences, but not with single-self, expected utility preferences; (2) The reflection effect is compatible with multiple-selves, expected utility preferences.

But there are some forms of gain-loss asymmetry that are incompatible with single-self preferences. More precisely, we decompose a reflection effect into two component effects, namely a probability switch and a translation. Either component is incompatible with single-self, expected utility preferences, while compatible with multiple-selves, expected utility preferences. But the probability switch effect is compatible with single-self, non-expected utility preferences, whereas the translation effect violates single self preferences.

The paper extends the analysis to cover the roles of wealth and of the amount of money at stake in decisions under uncertainty, and their implications for preference theory.

JEL Classification Numbers: D11, D81

# A Price Competition Game

# under Free Entry

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## Abstract

This study builds a game of Bertrand-like price competition in a market with free entry. Under the assumption of a standard U-shaped average cost curve, it demonstrates that even if the number of sellers is small, a long-run competitive outcome can be supported as a Nash equilibrium. This game provides unifying treatments to the standard Bertrand equilibrium, the long-run competitive outcome, Demsetz's equilibrium as well as other types of equilibria that have not been known in the existing literature.

# The Stability of Networks with Direct and Indirect Connections

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## Abstract

This paper discusses the nature of optimal and stable networks of the link formation game. Players are directly or indirectly connected in each network, and players' incentive to form new links depends upon the relative importance of direct and indirect links. We examine the value of networks for each player by employing the Shapley value. We will then illustrate the specific forms of the stable and optimal networks. Especially, we will show that characteristic networks such as the star or circle form networks can be both optimal and stable.

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