Extended Abstract

Simple Complexity from Imitation Games

by

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Gilboa and Zemel (1989) (henceforth GZ) established that certain computational problems related to Nash equilibrium of two player games are **NP**-hard. Connitzer and Sandholm (2003) (henceforth CS) have recently proved refinements of these results using different methods. In this paper we establish refinements of the results of GZ using imitation games, which are a class of two player game studied in McLennan and Tourky (2004). In comparison with GZ and CS, our arguments are simple and direct.

A *decision problem* is a computational problem for which the desired answer in either YES or NO. A computational problem is in **NP** if there is a polynomial time procedure that has a positive probability of establishing that the answer is YES when that is the case. A problem P is **NP**-*hard* if any other **NP** problem Q can be *polynomially reduced* to P, by which we mean that there is a polynomial time procedure for transforming an input for Q into an input for P with the same answer. A problem is **NP**-complete if it is both in **NP** and **NP**-hard. There are many known **NP**-complete problems, including CLIQUE (Given a simple undirected graph, is there a set of k vertices with an edge between each pair in the set?) and SAT (Given a conjunction of disjunctions of primitive propositions $P, Q, \ldots, R$ and their negations, is there a vector of truth values for $P, Q, \ldots, R$ such that the conjunction of disjunctions is true?)

The computational difficulty of computing a Nash equilibrium is unknown, but several related decision problems were shown by GZ to be **NP**-hard, including "Is there a second Nash equilibria?" Among other reductions, GZ obtain their results by giving polynomial reductions of the problems of interest to CLIQUE, while CS reduce to SAT. Their constructions are complex, with incentives that are far from obvious.

An *imitation game* is a two player normal form game in which the two agents' sets of pure strategies are the same and the second agent's payoff is 1 if she plays the same pure strategy as the first agent and 0 otherwise. The study of this class of games was initiated in McLennan and Tourky (2004), where we showed that, in spite of their simplicity, they capture the full computational complexity of general two person games. Here we give imitation games for which the problems studied by GZ and CS reduce, by straightforward arguments, to CLIQUE. In addition to providing simpler proofs of these results, this work lends further support to the contention that imitation games are a useful setting for studying complexity issues related to two person games because such issues tend to have intuitively simple and straightforward representations in this context.