EXISTENCE AND MULTIPLICITY OF PERIODIC SOLUTIONS FOR A SYSTEM OF AUTONOMOUS LOTKA-VOLTERRA TYPE EQUATIONS WITH DELAY

NORIMICHI HIRANO

1. Abstract

In this talk, we consider the existence and multiplicity of nonstationary periodic solutions of autonomous delay differential equations of Lotka-Volterra type

$$(1.1)\quad \begin{cases} \dot{u}_1(t) = & u_1(t)(r_1 - a_{11}u_1(t-\tau) - a_{12}u_2(t-\tau)\cdots - a_{1n}u_n(t-\tau)) \\ \dot{u}_2(t) = & u_2(t)(r_2 - a_{21}u_1(t-\tau) - a_{22}u_2(t-\tau)\cdots - a_{2n}u_n(t-\tau)) \\ & \cdots\cdots \\ \dot{u}_n(t) = & u_n(t)(r_n - a_{n1}u_1(t-\tau) - a_{n2}u_2(t-\tau)\cdots - a_{nn}u_n(t-\tau)) \end{cases}$$

where $n \geq 1, \tau > 0$, $\{r_i\}_{i=1}^n \subset \mathbb{R}$, $\{a_{ij}\}_{i,j=1}^n \subset \mathbb{R}$ and $\dot{u}_i = du_i/dt$ for each $i = 1, \cdots, n$. A broad class of problems in mathematical biology, economics and mechanics are described in the form above with initial conditions

$$(1.2)\quad \begin{cases} u_i(s) = \varphi_i(s), & s \in [-\tau, 0], \quad \varphi_i(0) > 0, \\ \varphi_i \in C([-\tau, 0]) & , \qquad i = 1, 2, \cdots, n. \end{cases}$$

In case $n = 1$, the problem (1.1) is known as delay logistic equation:

$$(\mathrm{L})\qquad \dot{u} = \alpha u(t)(1 - u(t-\tau)),$$

The existence and multiplicity of solutions of delay logistic equation has been investigated by many authors(cf. Hale[2] and Goparlsamy[1]). Our purpose in this talk is to show a new approach to this problem and establish the existence and multiplicity of solutions of problem (1.1). In our approach, we will work on the space of periodic functions instead of considering the initial value problem and we make use of the S^1-degree theory to show the existence and multiplicity of problem (1.1).

References

[1] K. Goparlsamy. "Stability and Oscillations in Delay Differential Equations of Polulation Dynamics". Kluwer Academic Publishers (1992).

[2] Jack Hale. "Theory of Functional Differential Equations". Springer-Verlag (1976).

Department of Mathematics, Graduate School of Envieronment and Information Sciences, Yokohama National University, Tokiwadai, Hodogayaku, Yokohama, Japan