On differentiability and bifurcation

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Abstract. For a function acting between Banach spaces, we recall the notions of Hadamard and w-Hadamard differentiability and their relation to the common notions of Gâteaux and Fréchet differentiability. We observe that even for a function \( F: H \to H \) that is both Hadamard and w-Hadamard differentiable but not Fréchet differentiable at 0 on a real Hilbert space \( H \), there may be bifurcation for the equation \( F(u) = \lambda u \) at points \( \lambda \) which do not belong to the spectrum of \( F'(0) \). We establish some necessary conditions for \( \lambda \) to be a bifurcation point in such cases and we show how this result can be used in the context of partial differential equations such as

\[-\Delta u(x) + q(x)u(x) = \lambda e^{-|x|} \tanh |x| u(x)^4 \quad \text{for} \quad u \in H^2(\mathbb{R}^N)\]

where this situation occurs.