

On differentiability and bifurcation

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Abstract. For a function acting between Banach spaces, we recall the notions of Hadamard and w-Hadamard differentiability and their relation to the common notions of Gâteaux and Fréchet differentiability. We observe that even for a function $F: H \rightarrow H$ that is both Hadamard and w-Hadamard differentiable but not Fréchet differentiable at 0 on a real Hilbert space H , there may be bifurcation for the equation $F(u) = \lambda u$ at points λ which do not belong to the spectrum of $F'(0)$. We establish some necessary conditions for λ to be a bifurcation point in such cases and we show how this result can be used in the context of partial differential equations such as

$$-\Delta u(x) + q(x)u(x) = \lambda e^{-|x|} \tanh(e^{|x|} u(x)) \quad \text{for } u \in H^2(\mathbf{R}^N)$$

where this situation occurs.