

## Direct and indirect connections, the Shapley value, and network formation

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**Abstract.** This paper discusses the nature of optimal and stable networks in the link formation game. Players are directly or indirectly connected in each network, and players' incentives to form new links depend upon the relative importance of these links. In this paper, instead of introducing a production function for each direct or indirect link to determine a player's payoffs and the network value, we define the value of each network (and its sub-networks) directly, and determine players' payoffs according to the Shapley value. To make the analysis tractable, we pay a special attention to the convexity or concavity of the underlying networks, and in another model, study how the optimal and the pairwise stable networks change as costs of link formation change. We will show that special networks such as the star or circle form networks, as well as the complete network, can be both optimal and stable.

**Key words:** network formation, Shapley value, pairwise stability, convexity and concavity of a network, link cost