Adv. Math. Econ. 7, 1-32 (2005)



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## Some variational convergence results for a class of evolution inclusions of second order using Young measures

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Received: May 6, 2004 Revised: September 6, 2004

JEL classification: C61

Mathematical Subject Classification (2000): 49J40, 49J45, 46N10, 34G25

**Summary.** This paper has two main parts. In the first part, we discuss the existence and uniqueness of the  $W_E^{2,1}$ -solution  $u_{\mu,\nu}$  of a second order differential equation with two boundary points conditions in a finite dimensional space, governed by controls  $\mu, \nu$  which are measures on a compact metric space. We also discuss the dependence on the controls and the variational properties of the value function  $V_h(t,\mu) := \sup_{\nu \in \mathcal{R}} h(u_{\mu,\nu}(t))$ , associated with a bounded lower semicontinuous function h. In the second main part, we discuss the limiting behaviour of a sequence of dynamics governed by second order evolution inclusions with two boundary points conditions. We prove that (up to extracted sequences) the solutions stably converge to a Young measure  $\nu$  and we show that the limit measure  $\nu$  satisfies a Fatou-type lemma in Mathematical Economics with variational-type inclusion property.

**Key words:** Fatou lemma, value function, second order differential equation, second order differential inclusion, Young measure, fiber product.