

## Fixed point theorems in Hausdorff topological vector spaces and economic equilibrium theory

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**Abstract.** The aim of this paper is to develop fixed point theorems in Hausdorff topological vector spaces that are suitable for the purpose of economic equilibrium theory. The special concept we have used here is the “direction structure” that characterizes mappings in the economic theory, (preferences, excess demands, and the like,) adequately, and enables us to modify problems on mappings into those on a structure of the base set. Especially, since our mathematical generalization may directly be related to the continuity and/or convexity of individual preferences, we may obtain existence theorems of maximal points, Pareto optimal allocations, and price equilibria for Gale-Nikaido-Debreu abstract economies under quite natural conditions.

**Key words:** fixed point theorem, non-ordered preference, direction structure, Gale-Nikaido-Debreu theorem, market equilibrium