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Functional evolution equations governed by m-accretive operators

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Abstract. Let $r > 0$ be a finite delay and $\mathcal{C}_0 = \mathcal{C}_E([-r, 0])$ be the Banach space of continuous vector-valued functions defined on $[-r, 0]$ taking values in a separable reflexive Banach space E such that its strong dual E' is uniformly convex. We discuss the existence of strong solutions for an evolution equation of the form $\dot{u}(t) \in -A(t)u(t) + F(t, \tau(t)u)$ a.e in $[0, 1]$, where $u : [-r, 1] \to E$ is a continuous mapping from $[-r, 1]$ into E such that its restriction to $[-r, 0]$ is equal to $\varphi \in \mathcal{C}_0$, and its restriction to $[0, 1]$ is absolutely continuous, (i.e $u(t) = u(0) + \int_0^t \dot{u}(s)\, ds, \forall t \in [0, 1]$ with $\dot{u} \in L^1_E([0, 1])$), and satisfies the preceding inclusion, $A(t)$ is an m-accretive multivalued operator on $E, F : [0, 1] \times \mathcal{C}_0 \to E$ is a convex weakly compact valued, separately scalarly measurable and separately scalarly upper semicontinuous multifunction and $(\tau(t)u)(s) = u(t + s), \forall s \in [-r, 0]$. Some applications to the sweeping process (or Moreau's process) and Optimal Control involving Young measures are also presented.

Key words: accretive operator, ball-compact, functional differential inclusion, maximal monotone operator, multifunction, original control, relaxed control, relaxation, Young measure