

## An overview of turnpike theory: towards the discounted deterministic case <sup>3</sup>

M. Ali Khan<sup>1</sup> and Adriana Piazza<sup>2</sup>

<sup>1</sup> Department of Economics, The Johns Hopkins University, Baltimore, MD 21218, USA (e-mail: akhan@jhu.edu)

<sup>2</sup> Departamento de Matemática, Universidad Técnica Federico Santa María, Avda. España 1680, Casilla 110-V, Valparaso, Chile (e-mail: adriana.piazza@usm.cl)

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**Abstract.** In the last 5 years, there has been extensive work on the existence and characterization of solutions of undiscounted optimal programs in simple discrete-time models of the ‘choice of technique’ in development planning and of lumber extraction in the economics of forestry. In this expository essay, we present a unified treatment of the characterization results in two of these models. Furthermore, with an eye towards extensions to the discounted setting, we present the general theory, both with or without “smoothness hypotheses” on the felicity function, and in continuous-time and discrete-time taking special care to distinguish asymptotic convergence of optimal programs from their classical turnpike properties. We show how the general results do not translate directly to the particular toy models examined here, and thereby suggest open problems for a more universal theory.

**Key words:** Intertemporal resource allocation, discount factor, optimal programs, asymptotic convergence, classical turnpike theory, neighborhood turnpike theorem, development planning, forest management

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