

Tightness conditions and integrability of the sequential weak upper limit of a sequence of multifunctions

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Abstract. Various notions of tightness for measurable multifunctions are introduced and compared. They are used to derive results on the existence of integrable selections for the sequential weak upper limit of a sequence of multifunctions. Similar questions are examined for multifunctions with values in a dual space. Some results are particularized in the single-valued case, and applications to the multidimensional Fatou Lemma, both in the primal and in the dual space, are derived. This is achieved under conditions weaker than or non comparable to L^1 -boundedness.

Key words: Tightness conditions, upper limit of a sequence of multifunctions, measurable selection, integrable selection, Fatou's Lemma in several dimensions